

# Environmental policy and speculation on markets for emission permits<sup>1</sup>

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<sup>1</sup>We are grateful to co-editor Arun Malik and two anonymous referees, as well as to Parkash Chander, Jacques Drèze, Alphonse Magnus, Enrico Minelli and Bert Willems for useful comments on earlier versions of the paper. Financial support from the European Community's Human Potential Programme under contract HPRN-CT-2002-00232 (MICFINMA) is acknowledged by Paolo Colla and from the Institut Europlace de Finance (EIF) by Paolo Colla and Marc Germain.

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## Abstract

Tradable emission permits share many characteristics with financial assets. As on financial markets, speculators are likely to be active on large markets for emission permits such as those developing under the Kyoto Protocol. We show that the presence of speculators on a market for emission permits affects the price of these permits when firms are risk averse. Since speculators help firms hedging the risk stemming from uncertain future demand, their entry reduces permits expected returns as well as their volatility. Moreover, we characterize the optimal environmental policy by the agency setting the total amount of permits to issue. Whenever the agency is sufficiently risk tolerant, speculators improve aggregate welfare by fostering firms production. On the other hand, in the presence of a moderately risk averse agency the increase in production volatility induced by speculators negatively affects social welfare.

**Keywords:** tradable emission permits, speculation, risk aversion, uncertainty, pollution control.

# Introduction

The interest in markets for emission permits has increased worldwide after the signature of the Kyoto Protocol in December 1997. Although emission constraints will only be binding from 2008 onwards, the embedded so-called “flexible mechanisms” for reducing greenhouse gas emissions are already giving rise to the largest markets for emission permits. The European market for carbon dioxide emission permits –launched in January 2005– as well as other national or regional initiatives, are paving the way towards a global market. Moreover, the implementation of emission permits markets is currently under development or consideration for the control of other air pollutants (such as SO<sub>2</sub>, NO<sub>x</sub>, VOC, etc.) in numerous countries (see e.g. Stavins [17]). The design of most of these schemes is inspired by the American experiences with such markets, especially the Acid Rain Program.

Emission permits share many characteristics with financial assets. An emission permit (also often called ‘allowance’) allows a regulated agent to emit a specified amount of a certain pollutant during a given period of time (e.g. 1 ton of sulfur dioxide or carbon dioxide in the year 2006). The permits are virtual assets: an agent holds a permit if this permit is registered on the account of that agent by the environmental agency. Hence, emission permits are perfect substitutes and their trade entails neither transportation nor inventory costs. Such characteristics are favorable to the entry on the market of other agents than regulated polluters, typically speculators.

Analyses of the US Acid Rain Program (see e.g. Schmalensee et al. [14]) or of the emerging European market for carbon dioxide emission permits (see e.g. Convery and Redmond [5]) reveal the presence of such agents on these markets. In fact, numerous financial institutions are active on the European market and hedge funds are expected to enter soon. At present, the number of market participants is already very large and predicted to sharply increase when all permits accounts become operational in the Eastern European countries. Moreover, although permits are currently traded at different places, market analysts forecast that trades will soon be performed on an unique centralized exchange. This would increase the transparency of the market and ease the access to it. Since, in addition, it is particularly

simple to open a permits account by the environmental agency<sup>1</sup>, the number of players other than regulated firms is likely to be very large.

Nevertheless, to our knowledge, the literature on the tradable emission permits instrument has so far ignored the presence of these agents. Although they focus on the microstructure on markets for emission permits, the contributions by Germain et al. ([8] and [9]) analyze only the role of intermediaries and show how a price-driven market may lead to a strictly positive spread. They leave aside the issue of speculation.

Analyzing speculation on markets for emission permits is particularly relevant because speculators have an impact on the equilibrium permits price and, consequently, on firms' investment/production choices. Speculators stand ready to accommodate the excess demand of permits stemming from firms. As such speculators serve as market makers for pollution permits, and the equilibrium price therefore includes a premium for holding inventories firms are willing to unload.<sup>2</sup> Such a change in the price signal would in turn affect firms investment and production decisions. Therefore, when the environmental agency balances the environmental quality with the cost for the firms of reducing pollution, it should account for the impact of speculators.

In terms of methodology, we introduce risk averse polluting firms which have to decide on the amount of capital (or abatement) under uncertainty. This is similar to the approach followed by Baldursson and von der Fehr [2] who introduce risk averse firms and analyze the performance of the tradable permits instrument with respect to the tax instrument. Among other things, they show that accounting for risk aversion tends to increase the relative performance of taxes. We make the additional assumption of constant absolute risk aversion in order to get closed form solutions and enrich the framework in three directions: we account for repeated permits trading rounds, we introduce risk averse speculators and we compute the optimal amount of emission permits to be issued by the environmental agency. Although we also take risk aversion into account, our paper differs from Baldursson and von der Fehr

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<sup>1</sup>Agents can manage their accounts via the internet, just like for internet banking.

<sup>2</sup>In a different context, Bernardo and Welch [3] consider the interaction between a pool of investors facing potential liquidity shocks and a sector of speculators absorbing unwanted inventories.

[2] both in its motivation and modelling framework.<sup>3</sup>

The main results of our paper are related to these aspects as follows. The uncertainty firms face when choosing their level of capital makes them willing to sell (part of) their emission permits during the first trading round. Once uncertainty is resolved and capital has been allocated, firms purchase back the permits at the second round. Speculators hold positive inventories of permits between the two dates and earn positive expected returns as compensation for their risk bearing activity. The analysis reveals that social welfare depends on the risk-bearing capacity of the market (which in turn depends on the risk attitude of the market participants, i.e. firms and speculators) as well as on the regulator's risk attitude.

The paper is organized as follows. In Section 1 we present the baseline model where firms only are active in the emission permits market. We analyze the equilibrium together with the policy pursued by the environmental agency in setting the optimal amount of allowances to be issued. In section 2 we generalize our model allowing speculators to trade in the permits market and highlighting their impact on the equilibrium outcome. The robustness of our analysis to changes in the modelling framework are further discussed in Section 3, together with the policy implications. Finally, in Section 4 we summarize our findings and present possible extensions to the current analysis.

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<sup>3</sup>Indeed, our aim is to analyse the impact of speculation on permits markets as well as on environmental policy. Moreover Baldursson and von der Fehr [2] use a more general formalization with respect to the objectives of the firms and consider uncertainty both at the aggregate (so called 'extraneous' risk) and firm-specific level. On the other hand, they do not consider non-polluting agents, i.e. speculators, and do not modelise the behaviour of the environmental agency. The differences between their formalization and ours explain some important differences in the results, for example on the role of the permits allocation between firms.

# 1 Emission permits and environmental policy

## 1.1 Modelling framework

We consider two types of agents: firms and an environmental agency. The agency is in charge of defining a total amount of emission permits and of (freely) allocating them to firms. We assume for the time being that the agency has already defined the total amount of permits denoted by  $S_0$ . The behavior of the agency, leading to the definition of the optimal amount of permits by taking into account the damage costs due to emissions of pollutants, is analyzed in subsection 1.3.

There is a continuum of firms with measure  $n_F$ . Each firm, indexed by  $i$ , produces the good  $y$  whose price is normalized to 1. All firms have the same Cobb-Douglas constant returns to scale production function

$$y_i = \theta k_i^\alpha e_i^{1-\alpha} \quad \text{with } 0 < \alpha < 1, \quad (1)$$

where  $k_i$  denotes the level of capital employed by firm  $i$  and  $e_i$  is the amount of emissions used by the same firm. The parameter  $\theta$  represents a shock to the aggregate demand. This shock affects all firms in the same way and is normally distributed<sup>4</sup> with mean  $\mu > 0$  and variance  $\sigma^2$ .

Firms purchase capital at the exogenous price  $r > 0$ . Moreover, when a firm emits pollutants, it must hold an amount of permits which is not lower than the level of emissions. Each firm freely receives from the environmental agency an amount of emission permits  $s_{i0}$  –which can differ across firms– and may sell some of its permits to –or purchase some additional ones from– other firms active in the permits market. Such trades take place at any moment after allowances have been allocated. For simplicity, we assume the permits market opens at two different trading periods indexed by  $t$  ( $t = 1, 2$ ) and let  $p_t$  denote the unit price of allowances during round  $t$ .

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<sup>4</sup>Under normality assumption, the demand shock parameter –and consequently revenues from product sales– might take negative values. However as pointed out in Vives [19] one can make the probability of negative prices (or output) arbitrarily small by choosing high values for  $\mu$  and low values for  $\sigma^2$ .

Firm  $i$  profit reads as

$$\pi_{Fi} = \pi_F(k_i, s_{i1}, e_i; \theta) = \theta k_i^\alpha e_i^{1-\alpha} - rk_i - p_1(s_{i1} - s_{i0}) - p_2(s_{i2} - s_{i1}), \quad (2)$$

where  $s_{it}$  ( $t = 1, 2$ ) is the amount of permits held by firm  $i$  at the end of trading period  $t$ , i.e. firm  $i$  inventory of permits at period  $t$ . Therefore firm  $i$  purchases (resp. sells) allowances at the trading round  $t$  if  $s_{it} - s_{it-1} > 0$  (resp.  $s_{it} - s_{it-1} < 0$ ).

From eq. (2) each firm total profit is composed of the revenues from the sales of the product,  $y_i$ , the cost of capital and the cost (resp. benefits) of the net permits purchases (resp. sales). As long as permits prices are strictly positive, the requirement that each firm must hold an amount of permits greater or equal to its emissions level ( $s_{i2} \geq e_i$ ) will hold with equality, and we set  $s_{i2} = e_i$  in eq. (2).

Firms are risk-averse as captured by a constant absolute risk-aversion (CARA) function defined over their profits, and we let  $\tau_F > 0$  denote the absolute risk-tolerance coefficient (i.e. the inverse of the absolute risk-aversion parameter) taken to be identical for all firms. Assuming CARA investors together with normality of the underlying return distribution is very common in finance thanks to its analytical tractability. Many portfolio problems are cast within the so-called CARA-Gaussian setup because the certainty equivalent of a normal random variable is linear in its mean and variance when the utility function is exponential (see for example Brunnermeier [4]). Indeed, the idea of mean-variance preferences is a cornerstone for modern portfolio theory (see Markowitz [12]). When dealing with firms, the determination of the corporate risk attitude is a non-trivial issue as shown in Smith [15]. Corporate decision making reflects (or should reflect) the preferences of several agents such as shareholders and/or managers. Moreover, in the former case firms' risk preferences are affected by the degree of shareholding diversification. However, coming up with a model that explicitly keeps into account all these dimensions of corporate risk attitude is beyond the scope of our paper, and we directly employ a CARA utility function for firms as well as the agency (see subsection 1.3).

The sequence of decisions is as follows.<sup>5</sup> In period 0, the agency issues and allocates to

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<sup>5</sup>Our timeline is close to the one employed by Baldursson and von der Fehr [2] when dealing with forward

firms the total amount of permits  $S_0 = \int_0^{n_F} s_{i0} di$ . In period 1, firms face uncertainty on the demand shock parameter  $\theta$  and simultaneously choose the amount of capital,  $k_i$ , and the emission permits inventory level,  $s_{i1}$ . In period 2, firms decide on their amount of emissions,  $e_i$ , and accordingly trade permits,  $e_i - s_{i1}$ , after the demand shock has realized. This sequence of decisions is illustrated in Figure 1. We now solve the model by backward induction.

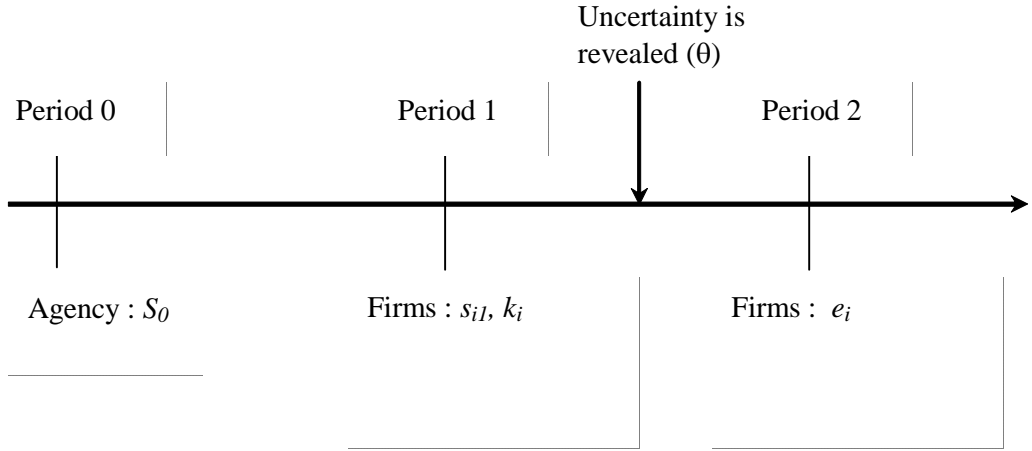


Figure 1: Sequence of decisions (baseline model)

## 1.2 Emission permits market

### 1.2.1 Second round equilibrium

At time  $t = 2$  there is no uncertainty on the demand shock parameter  $\theta$ . Each firm maximizes profits with respect to the emissions level, given its first period choices for  $k_i$  and  $s_{i1}$ :

$$\max_{e_i} \pi_F(k_i, s_{i1}, e_i; \theta), \quad (3)$$

with  $\pi_F(\cdot)$  as in eq. (2). The first order condition gives the following demand schedule for permits chosen by the firm<sup>6</sup>

$$e_i(p_2) = \left[ \frac{(1 - \alpha)\theta}{p_2} \right]^{\frac{1}{\alpha}} k_i. \quad (4)$$

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trading (see their section 5.2).

<sup>6</sup>All the computations (including the proofs of the following propositions) are detailed in a separate appendix to this paper available upon request.

Firm  $i$  net demand for allowances is then  $e_i(p_2) - s_{i1}$ .

The permits market clearing condition in period 2 reads as follows:

$$\int_0^{n_F} (e_i(p_2) - s_{i1}) di = 0, \quad (5)$$

or equivalently  $\int_0^{n_F} e_i(p_2) di = \int_0^{n_F} s_{i1} di$ , i.e. firms' emissions are equal to the stock of permits held by firms after the first trading round. As is clear, this stock must be equal to the amount of permits the agency distributes to firms at time 0 and eq. (5) reduces to  $\int_0^{n_F} e_i(p_2) di = S_0$ . Using the latter condition and firm  $i$  demand schedule in eq. (4) yields<sup>7</sup> the second period permits price

$$p_2 = (1 - \alpha) \left( \frac{K}{S_0} \right)^\alpha \theta, \quad (6)$$

where  $K$  denotes the aggregate level of capital, i.e.  $K = \int_0^{n_F} k_i di$ . Thus, in equilibrium firm  $i$  emissions are given by

$$e_i = \frac{S_0}{K} k_i. \quad (7)$$

### 1.2.2 First round equilibrium

At time  $t = 1$ , firms face uncertainty about the date 2 product demand. Each firm solves  $\max_{k_i, s_{i1}} E(u_F(\pi_{Fi}))$  subject to the permit price and the emissions level prevailing at date 2, where  $u_F(\cdot)$  denotes firm  $i$  utility function. Substituting the equilibrium values for  $p_2$  and  $e_i$  (see eqs. (6) and (7)) into the expression for  $\pi_{Fi}$  as given in eq. (2) yields

$$\pi_{Fi} = \pi_F(k_i, s_{i1}; \theta) = \frac{\theta K^\alpha}{S_0^\alpha} \left( \alpha \frac{S_0}{K} k_i + (1 - \alpha) s_{i1} \right) - rk_i - p_1(s_{i1} - s_{i0}). \quad (8)$$

Since the demand shock  $\theta$  is normally distributed with mean  $\mu$  and variance  $\sigma^2$ , then profits in eq. (8) are also normally distributed. As is known (see for example Lintner [11]), optimization

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<sup>7</sup>Since  $\theta$  is normally distributed,  $p_2$  could in principle be negative. However, as mentioned in footnote 4, we assume that  $\mu$  (resp.  $\sigma^2$ ) is sufficiently high (resp. low) so that the probability of such a situation can be neglected. Assuming the demand shock  $\theta$  to be lognormally distributed is another option to deal with non-negative output and prices. However, to our knowledge this case can't be analytically characterized. On this point, it could be noted that the normal distribution provides a reasonable approximation to the lognormal when the coefficient of variation  $\sigma/\mu$  is sufficiently small (see Johnson et al. [10]).

of a CARA utility function defined over a normal random variable allows to write firm  $i$  maximization problem as

$$\max_{k_i, s_{i1}} E(u_F(\pi_{Fi})) = \max_{k_i, s_{i1}} \mu_F(k_i, s_{i1}) - (2\tau_F)^{-1} \sigma_F^2(k_i, s_{i1}), \quad (9)$$

where  $\mu_F(\cdot)$  and  $\sigma_F^2(\cdot)$  denote respectively the mean and variance of firm  $i$  profits. Although firms are price taker and face constant returns to scale, note that this problem has an interior solution since uncertainty and risk aversion make firms' expected utility a concave function in its arguments as noted by Sandmo [13].

The first order conditions of the problem (9) lead to the following demand schedule for capital

$$k_i(p_1) = \frac{\alpha p_1 S_0}{(1 - \alpha) r n_F}. \quad (10)$$

Relation (10) means that the optimal amount of capital chosen by firms increases in the total amount of permits allocated ( $S_0$ ) as well as the price of the other input ( $p_1$ ) and decreases in its own price ( $r$ ). Note that the demand for capital is identical across firms since they have the same technology and risk aversion. In the remainder we therefore use  $k$  instead of  $k_i$ .

At the aggregate level, since there is a continuum of firms of measure  $n_F$ , the total amount of capital is then  $K = n_F k$ , i.e.

$$K(p_1) = \frac{\alpha p_1 S_0}{(1 - \alpha) r}. \quad (11)$$

Turning to permit inventories, optimality conditions yield

$$s_{i1}(p_1) = \frac{\tau_F}{(1 - \alpha)^{2(1-\alpha)} \sigma^2} \left(\frac{r}{\alpha}\right)^{2\alpha} \frac{1}{p_1^\alpha} \left[ (1 - \alpha)^{1-\alpha} \left(\frac{\alpha}{r}\right)^\alpha \mu - p_1^{1-\alpha} \right] - \frac{\alpha S_0}{(1 - \alpha) n_F}. \quad (12)$$

The demand schedule in eq. (12) shows that the amount of permits held by firm  $i$  in period 1 is decreasing in the amount of permits issued ( $S_0$ ) as well as the permit price ( $p_1$ ), increasing in the expected demand shock ( $\mu$ ) and decreasing in its uncertainty ( $\sigma^2$ ). As previously noted for the optimal amount of capital,  $s_{i1}$  is identical across firms, i.e. it does not depend on the initial amount of allowances received  $s_{i0}$ . To ease notation, in the remainder we simply write  $s_1$  instead of  $s_{i1}$ .

The first period market clearing condition is

$$\int_0^{n_F} (s_1(p_1) - s_{i0}) di = 0. \quad (13)$$

Let  $\tau$  denote the risk-bearing (or risk-tolerance) capacity of the market, i.e.

$$\tau = \tau_F n_F. \quad (14)$$

As is clear, the market risk-bearing capacity is an increasing function of each firm degree of risk-tolerance as well as their measure.

Using the permit demand schedule in eq. (12) and the market risk-bearing capacity (see eq. (14)), the first period permits price solves the implicit function

$$S_0 = \frac{\tau}{\sigma^2} \left[ \frac{(1-\alpha)r}{\alpha p_1} \right]^\alpha \left[ \mu - \left( \frac{r}{\alpha} \right)^\alpha \left( \frac{p_1}{1-\alpha} \right)^{1-\alpha} \right]. \quad (15)$$

Equation (15) does not lend itself to explicitly compute<sup>8</sup> the equilibrium price  $p_1$ . However, the existence and uniqueness of such an equilibrium price<sup>9</sup> on the positive half-line are addressed in the following

**Proposition 1** *For any finite positive  $S_0$ , there exists a unique equilibrium price  $p_1 \in \mathfrak{R}^{++}$ .*

We further establish some properties of the equilibrium price by means of

**Proposition 2** *The equilibrium price  $p_1$*

- a) decreases with  $S_0$ ;*
- b.i) decreases with  $\sigma^2$  and*
- b.ii) increases with  $\mu$ ;*
- c) increases with  $\tau$*

According to Proposition 2.a the date 1 permit price decreases when the total supply of allowances increases. This is rather intuitive and stems from the fact that the demand curve for permits is downward sloped, as previously noted (see eq. (12)).

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<sup>8</sup>Noteworthy the way permits are shared among firms is irrelevant and it does not play any role in the model solution. This is in sharp contrast with Baldursson and von der Fehr [2] and follows from the fact that transfers (permits allocation) appear only linearly in our linear-quadratic setting.

<sup>9</sup>If the agency allocates the same amount of permits to all firms, then there would be no trade in equilibrium among firms. This does not imply that allowances have no price at date 1. The market clearing price in eq. (15) supports the no trade equilibrium where  $s_1 = \frac{S_0}{n_F}$  (see for example Debreu [6]).

In order to understand the dependence of the equilibrium price  $p_1$  on the parameters describing the demand shock (Proposition 2.b) note from the expression for the date 2 permit price in eq. (6) that, other things equal: 1) larger values for the expected demand shock  $\mu$  generate higher expected permit prices at date 2, since energy consumption –from which pollution follows– becomes more valuable to firms as a response to the larger demand for the final output, and 2) larger values for the uncertainty of demand shock  $\sigma^2$  translate into more volatile permit prices at date 2, reflecting the increased variability in the final product market. In the former case (higher  $\mu$ ) firms anticipate the increase in  $p_2$  which induces them to hold more of their allowances at date 1, thus decreasing the supply of permits to the market and raising their date 1 price. In the latter case (higher  $\sigma^2$ ) firms anticipate the variability in date 2 price and are willing to decrease their date 1 inventories in order to temper out profits, i.e. reduce them in good states (high realization of  $\theta$ ) and increase them in bad states (low realization of  $\theta$ ). The larger supply by firms thus decreases  $p_1$ .

An increase in firms' risk tolerance,  $\tau_F$ , would reduce the negative effect that the uncertainty about date 2 price movements exerts on firms' expected utility. Therefore at date 1 firms are more willing to hold on to their permits, and lower supply would move  $p_1$  up. A similar effect is triggered by an increase in the measure  $n_F$ , since the negative impact of uncertainty about demand conditions is shared among more firms, thus again increasing the permit price at the first trading round. Proposition 2.c summarizes these implications.

### 1.2.3 Comparative statics: expected returns, volatility and capital

We now focus on studying the dynamics of permits prices. Let  $R = (p_2/p_1) - 1$  denote the (net) return earned on permits between the two trading rounds.  $E(R)$  and  $V(R)$  denote expected returns and volatility respectively.

**Proposition 3** *In equilibrium:*

a) *expected returns are strictly positive, i.e.  $E(R) > 0$*

*Moreover both expected returns and volatility*

b.i) *increase with  $S_0$ ;*

*b.ii) increase with  $\sigma^2$ ;*

*b.iii) decrease with  $\tau$ .*

*Finally*

*c) volatility decreases with  $\mu$*

Proposition 3.a states that permits prices are expected to rise through time. Holding allowances between the two dates entails some risk, since uncertainty is resolved after the first trading round and firms are risk-averse. It therefore follows that permits trade at a premium, i.e. the (strictly positive) difference between  $E(p_2)$  and  $p_1$ , or equivalently they guarantee positive expected returns to their holders.

In order to understand the rationale behind the other predictions of Proposition 3, note that substituting aggregate capital (see eq. (11)) into the date 2 price (see eq. (6)) yields the following

$$1 + R = \left(\frac{\alpha}{r}\right)^\alpha \left(\frac{1 - \alpha}{p_1}\right)^{1-\alpha} \theta, \quad (16)$$

which say that the return on allowances is inversely related to the first trading round price. By means of eq. (16) expected returns and volatility are

$$E(R) = \left(\frac{\alpha}{r}\right)^\alpha \left(\frac{1 - \alpha}{p_1}\right)^{1-\alpha} \mu - 1, \quad (17)$$

$$V(R) = \left(\frac{\alpha}{r}\right)^{2\alpha} \left(\frac{1 - \alpha}{p_1}\right)^{2(1-\alpha)} \sigma^2 \quad (18)$$

and therefore  $E(R)$  and  $V(R)$  depend on the parameters under consideration, i.e.  $\tau$ ,  $\mu$ ,  $\sigma^2$  and  $S_0$ , through the date 1 equilibrium price  $p_1$ .<sup>10</sup> Since  $E(R)$  and  $V(R)$  are inverse functions of  $p_1$  according to eqs. (17) and (18), changes in the underlying parameters that affect the date 1 equilibrium price in one direction will exert the opposite effect on expected returns and volatility. Then the comparative statics on  $E(R)$  and  $V(R)$  in Proposition 3.b hinge on the very same reasoning offered for Proposition 2.b. For example, when fewer permits are issued by the agency, firms are more willing to hold on their inventories so that

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<sup>10</sup>For the dependence of  $V(R)$  w.r.t.  $\sigma^2$  one must in addition take into account the direct effect of  $\sigma^2$ .

the date 1 price increases. This reduces the compensation required for holding permits both in its level  $E(R)$  and variability  $V(R)$  as in Proposition 3.b.i.

Similarly to the reasoning offered for Proposition 3.b.i, when the level of uncertainty increases (Proposition 3.b.ii) or the market risk-bearing capacity decreases (Proposition 3.b.iii), permit inventories become more risky and the compensation for holding them necessarily increases. Moreover, as the demand uncertainty or the market risk-aversion rise, the volatility of  $p_2$  increases and so does return volatility.

The intuition behind Proposition 3.c is as follows. Note from eq. (6) that the expected demand shock  $\mu$  affects the date 2 price volatility only through the term  $K^{2\alpha}$ . Thus, the dependence of  $V(p_2)$  on the aggregate capital is positive but less than quadratic since  $\alpha \in (0, 1)$ . By eq. (11) aggregate capital is linear in  $p_1$ , i.e. the price of the other input (see also Proposition 4 on this point). An increase in  $\mu$  will then raise both the first period price by Proposition 2.b.ii and the volatility of  $p_2$ . However the latter increase is less than the increase in  $p_1^2$  so that  $V(R) = V(p_2)/p_1^2$  decreases.

It is important to underline that our finding in Proposition 3.b.i could appear counter-intuitive for a very large level –possibly infinite– of initial permits  $S_0$ , i.e. when the environmental constraint linked to  $S_0$  is not binding or when there is no regulation at all. Then one might expect that  $p_1 = p_2 = 0$  and nil expected returns  $E(R) = 0$ .<sup>11</sup> Indeed, Proposition 3.b.i follows from the fact that (1) our model only applies to a regulated economy where a permits market exists (which implies that  $S_0$  is finite) and (2) there always exists a sufficient demand of permits to meet any large finite supply  $S_0$ .

To include the possibility that  $E(R) = 0$  in our model, one could introduce an explicit price for energy (besides the price of permits) or another constraint on the activity of firms (due for example to limits to the demand for the industry output or to available inputs). In such a framework the demand of permits by firms would remain upper bounded whatever the state of the world even if the price of permits tends to zero. Let  $S_b$  be this upper bound, then one may conjecture that  $E(R)$  would be monotonically increasing when  $S_0 < S_b$ , would

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<sup>11</sup>We are indebted to an anonymous referee for drawing our attention to this problem.

drop in the neighborhood of  $S_b$ , and would remain equal to 0 for  $S_0 > S_b$ .

Assessing the robustness of Proposition 3.b.i by extending our model in the way suggested above would certainly deserve attention. It will not be a trivial task however because one will lose the linear-quadratic framework that keeps the model tractable, i.e. one cannot neglect anymore the possibility of negative permits price in the second period. We thus leave this question to future research, and highlight that our results (in particular Proposition 3.b.i) are relevant when  $S_0$  is “small enough”, that is when the environmental constraint is really binding.<sup>12</sup>

After characterizing the equilibrium in the permits market, we turn to evaluate the effect of changes in the parameters on aggregate capital  $K = n_F k$ .

**Proposition 4** *In equilibrium the aggregate capital  $K$*

*a) increases (resp. decreases) with  $S_0$  over the interval  $(0, S_0^*)$  (resp.  $(S_0^*, +\infty)$ ) where*

$$S_0^* = \frac{\mu\tau}{2\sigma^2} \left[ \left( \frac{r}{\alpha} \right)^{1-2\alpha} \frac{2}{\mu} \right]^{\frac{\alpha}{1-\alpha}} ; \quad (19)$$

*b.i) decreases with  $\sigma^2$  and*

*b.ii) increases with  $\mu$ ;*

*c) increases with  $\tau$ .*

Recall from eq. (11) that the aggregate capital increases in the permit price  $p_1$ . Intuitively, this is so because when permits become more expensive, firms tend to substitute emissions

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<sup>12</sup>Indeed, one may argue that the objective of any market for emission permits is to effectively generate a scarcity on emissions in order to reduce them. This is clearly the case for well-known markets such as those created under the Acid Rain Program. Even the emerging or pilot markets in the context of greenhouse gas emission reductions are evolving in such a way that more and more stringent caps will be set. In fact, significant emission reductions are required in the near future, especially because world emissions are predicted to increase very much. All the efforts of the international community, supported by the work of the International Panel on Climate Change, go in such a direction.

with capital. Therefore Proposition 4.*b* and *c* directly follow from the behavior of  $p_1$  with respect to the relevant parameters  $\mu$ ,  $\sigma^2$  and  $\tau$ . On the other hand, the dependence of aggregate capital on the initial amount of permits  $S_0$  is less obvious. Indeed, from eq. (11) an increase in  $S_0$  proportionally raises the demand for capital (output effect). However, when more permits are allocated to firms,  $p_1$  decreases and firms tend to substitute capital with energy and  $K$  decreases (substitution effect). Therefore the overall effect is a priori ambiguous. As Proposition 4.*a* reveals, the output effect dominates the substitution effect whenever the amount of permits is sufficiently small, while the opposite holds true when  $S_0$  is large enough.

### 1.3 Optimal environmental policy

When defining the optimal amount of permits to issue and allocate to firms, the agency balances the social gains from reducing the total amount of emissions with the losses in production due to the constraint on emissions. The present subsection is devoted to this analysis.

We assume the agency maximizes the ‘green’ revenue of the industry, i.e. the aggregate production of firms less their consumption of capital and the damage costs due to pollution. For simplicity, we consider constant marginal damage costs. To be consistent with our formalization for firms’ preferences,<sup>13</sup> we also allow for constant absolute risk-aversion (CARA) on the part of the agency as captured by the absolute risk-tolerance coefficient  $\tau_A$ .

Since  $K = \int_0^{n_F} k_i di$  and in equilibrium  $S_0 = \int_0^{n_F} e_i di$ , the agency objective is defined in terms of the aggregate welfare

$$\theta K^\alpha S_0^{1-\alpha} - rK - \delta S_0, \quad (20)$$

where  $\delta S_0$  is the total damage associated with pollution level  $S_0$ . In equilibrium firms use the same amount of capital and permits (see eqs. (10) and (7) respectively). From the production function in eq. (1) one has that the first term in eq. (20) is the aggregate

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<sup>13</sup>See our discussion in subsection 1.1 when dealing with risk-averse firms.

production  $Y = \int_0^{n_F} y_i di = \theta K^\alpha S_0^{1-\alpha}$ . Letting  $\mu_A(\cdot)$  and  $\sigma_A^2(\cdot)$  denote the first two moments of the aggregate welfare, i.e.

$$\begin{aligned}\mu_A(S_0) &= E(Y) - rK - \delta S_0 \\ \sigma_A^2(S_0) &= V(Y)\end{aligned}\tag{21}$$

the problem of the agency can be written as follows:

$$\begin{aligned}\max_{S_0} W(S_0) &= \max_{S_0} \mu_A(S_0) - (2\tau_A)^{-1} \sigma_A^2(S_0) \\ \text{subject to eqs. (11), (15) and } S_0 &\geq 0.\end{aligned}\tag{22}$$

Let  $\hat{S}_0$  be a solution to the optimization problem (22) and  $W(\hat{S}_0)$  the corresponding welfare, i.e.  $W(\hat{S}_0) = \mu_A(\hat{S}_0) - (2\tau_A)^{-1} \sigma_A^2(\hat{S}_0)$ . The remainder of the analysis concerns the characterization of a non-negative solution to (22), as well as the comparative statics of the aggregate welfare with respect to the marginal damage  $\delta$ . The existence and uniqueness of a solution to the problem (22) are addressed in the following:

**Proposition 5** *The agency optimization problem admits a unique positive maximum  $\hat{S}_0 > 0$  for which  $W(\hat{S}_0) > 0$  if and only if*

$$\delta < \hat{\delta} = (1 - \alpha) \left[ \left( \frac{\alpha}{r} \right)^\alpha \mu \right]^{\frac{1}{1-\alpha}}.\tag{23}$$

In words, a necessary and sufficient condition for the socially optimal pollution level to be positive is that the marginal willingness to pay for the environment is not too big. Indeed, if marginal damage costs are large, i.e.  $\delta \geq \hat{\delta}$ , the agency chooses to issue and allocate no emission permits at all, so that production –and therefore pollution– will not take place. The role played by the marginal damage is further highlighted in the following:

**Proposition 6** *If  $\delta < \hat{\delta}$ , then  $\frac{d\hat{S}_0}{d\delta} < 0$  and  $\frac{dW(\hat{S}_0)}{d\delta} < 0$ .*

Proposition 6 asserts that –consistently with the intuition behind Proposition 5– the higher the marginal damage ( $\delta$ ), the lower the amount of permits issued and the lower the social welfare.

## 2 A setup with speculators

### 2.1 Introducing speculators

We now extend the previous analysis to introduce a third type of agents, namely speculators. Unlike firms, speculators do not produce or pollute and their profits result from trades only. There exists a continuum of identical speculators indexed by  $j$  and of measure  $n_S$ . We assume that speculators have no initial endowments of permits, i.e. all allowances issued by the agency are allocated to firms. Speculators profit from the price difference between the two trading rounds by purchasing (or short-selling) permits in the first trading period and unwinding their position in the second trading round. Hence their profit function is given by

$$\pi_{Sj} = \pi_S(x_{j1}, x_{j2}; \theta) = -p_2 x_{j2} - p_1 x_{j1}, \quad (24)$$

where  $x_{jt}$  is the purchase ( $x_{jt} > 0$ ) or sale ( $x_{jt} < 0$ ) of permits in trading period  $t$ , subject to  $x_{j1} + x_{j2} \geq 0$ , i.e. speculators should hold non-negative inventories in the last period. As long as allowances prices are strictly positive, speculators set  $x_{j2} = -x_{j1}$ .

We assume speculators are also risk-averse with CARA utility function defined over final profits  $\pi_{Sj}$  and denote by  $\tau_S > 0$  their absolute risk-tolerance coefficient assumed to be identical among them.

Figure 2 illustrates the sequence of decisions in the presence of speculators.

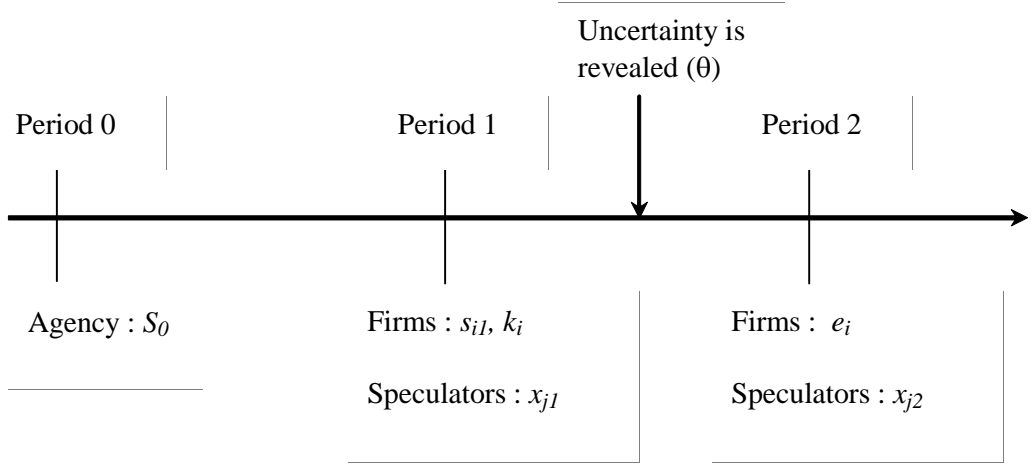


Figure 2: Sequence of decisions (extended model).

## 2.2 Emissions permits market

### 2.2.1 Second round equilibrium

At time  $t = 2$  there is no uncertainty on the demand shock parameter  $\theta$ . Firms maximize profits with respect to the level of emissions, given their first period choices for  $k_i$  and  $s_{i1}$ , so that their demand schedule for permits is given by eq. (4). At time 2 speculators unwind their date 1 position in the permits, thus ending up with null inventories at the second trading round, i.e.  $x_{j2} = -x_{j1}$ , where  $x_{j1}$  is chosen in period 1.

Accordingly, the permits market clearing condition in period 2 implies that firms' emissions are equal to the stock of permits held by firms *and* speculators after the first trading round. Thus eq. (5) becomes:

$$\int_0^{n_F} (e_i(p_2) - s_{i1}) di + \int_0^{n_S} -x_{j1} dj = 0. \quad (5')$$

Since firms' emissions must be equal to the amount of permits the agency distributes to firms at time 0, eq. (5') reduces to  $\int_0^{n_F} e_i(p_2) di = S_0$  which together with firm  $i$  demand schedule in eq. (4) yields the date 2 permits price as in eq. (6) and firms emissions as in eq. (7).

### 2.2.2 First round equilibrium

Since at date 2 speculators simply unwind their date 1 position, the expression for the equilibrium price  $p_2$  and the emission level  $e_i$  (see eqs. (6) and (7)) is unaltered by the presence of speculators. Therefore firms' demand schedules for capital and allowances are unchanged<sup>14</sup> and given by eqs. (10) and (12) respectively.

The speculators' problem is defined along the same lines as for firms. Each speculator solves  $\max_{x_{j1}} E(u_S(\pi_{Sj}))$ , where  $u_S(\cdot)$  denotes speculator  $j$ 's utility function. Since all speculators are identical,  $x_{j1} = x_1$  for all  $j$  and we drop the subscript  $j$  in the remainder. Given the equilibrium price  $p_2$  (see eq. (6)), profits  $\pi_S$  as defined in eq. (24) are normal with first two moments  $\mu_S(\cdot)$  and  $\sigma_S^2(\cdot)$ . Hence, the maximization problem becomes

$$\max_{x_1} E(u_S(\pi_{Sj})) = \max_{x_1} \mu_S(x_1) - (2\tau_S)^{-1} \sigma_S^2(x_1) \quad (25)$$

and the optimal demand (or inventory) of each speculator is

$$x_1(p_1) = \frac{\tau_S}{(1-\alpha)^{2(1-\alpha)} \sigma^2} \left(\frac{r}{\alpha}\right)^{2\alpha} \frac{1}{p_1^\alpha} \left[ (1-\alpha)^{1-\alpha} \left(\frac{\alpha}{r}\right)^\alpha \mu - p_1^{1-\alpha} \right]. \quad (26)$$

The speculators' demand schedule in eq. (26) is negatively sloped, i.e.  $x_1$  depends negatively on  $p_1$ . Moreover the dependence on the demand shock parameters  $\mu$  and  $\sigma^2$  is analogous to the one found for firms (see eq. (12)).

In the presence of speculators, the date 1 market clearing condition in eq. (13) becomes

$$\int_0^{n_F} (s_1(p_1) - s_{i0}) di + \int_0^{n_S} x_1(p_1) dj = 0, \quad (13')$$

and the market risk-bearing capacity (see eq. (14)) is replaced by

$$\begin{aligned} \tau' &= \tau_F n_F + \tau_S n_S \\ &= \tau + \tau_S n_S. \end{aligned} \quad (14')$$

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<sup>14</sup>Obviously, the presence of speculators does change the *level* of both capital and permit inventories through the effect on date 1 price.

As is clear, the market risk-bearing capacity is an increasing function of *both* agents' degree of risk-tolerance as well as their measure. The positive dependence exerted by the presence of speculators on the market risk-bearing capacity is crucial in determining the impact of speculators, as will become clearer in the remainder.

Using permit demand schedules (see eqs. (12) and (26)) and the market risk-bearing capacity (see eq. (14')), the first period permits price solves the implicit function (15) where  $\tau$  is now replaced by  $\tau'$ , i.e.

$$S_0 = \frac{\tau'}{\sigma^2} \left[ \frac{(1-\alpha)r}{\alpha p_1} \right]^\alpha \left[ \mu - \left( \frac{r}{\alpha} \right)^\alpha \left( \frac{p_1}{1-\alpha} \right)^{1-\alpha} \right]. \quad (15')$$

The uniqueness of a positive date 1 permit price (see Proposition 1) continues to hold in the presence of speculators as well as the comparative statics on  $p_1$  (see Proposition 2). Since the demand curves for both firms *and* speculators are downward sloped by eqs. (12) and (26), then  $p_1$  decreases with the total permit supply  $S_0$ . The dependence of the equilibrium price  $p_1$  on the parameters describing the demand shock is as follows. Subsection 1.2.2 reveals that firms reduce the supply of permits at date 1 since they anticipate the increase in the expected date 2 price induced by larger values for  $\mu$ . Higher expected date 2 prices mean more profit opportunities for speculators as measured by the expected price differential  $E(p_2) - p_1$ , thus increasing the speculators' incentives to demand permits at date 1. The combined effect of higher demand by speculators and lower supply by firms raises date 1 price and Proposition 2.b.ii follows. On the other hand, firms react to an increase in the demand shock uncertainty (as captured by higher values for  $\sigma^2$ ) by decreasing their date 1 inventories in order to temper out profits. At the same time, more volatile prices at the second trading round imply more uncertain profit opportunities for speculators. This negatively affects the speculators' willingness to buy permits at date 1. The overall effect of larger supply by firms and lower demand by speculators is a decrease in date 1 permit price (Proposition 2.b.iii). Finally Proposition 2.c is replaced by the following

**Proposition 2 .c')** *The equilibrium price  $p_1$  increases with  $\tau'$*

The intuition behind Proposition 2.c' is similar to the one offered for Proposition 2.c. An increase in the risk-tolerance of all speculators –either due to an increase in their measure or

in their individual risk bearing capacity— makes them more prone to buy permits from firms and thus raises the date 1 permit price.

It is of particular interest to identify the impact of the underlying parameters on expected returns since these measure the speculators' profit opportunities and thus affect their incentives to enter the emission permits market. The attention devoted to the impact of speculation on price fluctuations in financial markets by a wide range of agents, including authorities monitoring markets' behavior (see for example the recent UK FSA [18] report on hedge funds and their impact on market volatility) motivates the analysis of return volatility as well. As previously noted, the presence of speculators does not affect the expression neither for aggregate capital in eq. (11) nor for the date 2 price in eq. (6). It follows that expected returns and volatility are still given by eqs. (17) and (18) so that Proposition 3 continues to hold once part *c* is replaced by the following

**Proposition 3 .c')** *In equilibrium both expected returns and volatility decrease with  $\tau'$ .*

Expected returns are strictly positive according to Proposition 3.a which implies that speculators' trades are characterized by the following

**Proposition 8** *In equilibrium speculators buy permits at the first trading round, i.e.  $x_1 > 0$ .*

Proposition 8 says that speculators are ready to bear part of the risk firms face by holding the risky asset ( $x_1 > 0$ ) provided they are compensated for that. The compensation takes place through the (strictly positive) difference between  $E(p_2)$  and  $p_1$ , i.e. through positive expected returns. Thus in the presence of speculators there is trade in the permits market even if initial allowances are identical across firms (see also footnote 9).

Since speculators affect the demand for capital only through the date 1 permit price, then the comparative statics on aggregate capital in Proposition 4 continue to hold once  $S_0^*$  in eq. (19) is replaced by

$$S_0^* = \frac{\mu\tau'}{2\sigma^2} \left[ \left( \frac{r}{\alpha} \right)^{1-2\alpha} \frac{2}{\mu} \right]^{\frac{\alpha}{1-\alpha}} \quad (19')$$

and Proposition 4.c becomes

**Proposition 4 .c')** *In equilibrium the aggregate capital  $K$  increases with  $\tau'$ .*

An increase in the overall risk bearing capacity of speculators –either through an increase in  $\tau_S$  or  $n_{S-}$ – increases the permit price during the first trading round (see Proposition 2.c') and firms then substitute emissions with capital so that Proposition 4.c' follows.

### 2.2.3 The impact of speculators

By means of our theoretical model we can analyze how the entry of speculators in the market for emission permits affects equilibrium variables. As is clear, this is of primary interest from a policy perspective, and our model has sharp predictions in this respect. In fact, this amounts to compare the equilibrium in the baseline model of section 1 to the one of the extended model when speculators are active too. Thanks to Propositions 2.c', 3.c' and 4.c' we can evaluate such effects, summarized in the following

**Result:** Given the total amount of permits  $S_0$ , the entry of speculators in the emission permits market

- 1) decreases expected returns and volatility
- 2) increases aggregate capital,  $K$ , and production,  $Y$ .

Inspection of eq. (14') reveals that the presence of speculators acts through an increase in the market risk-bearing capacity. Overall, speculators help firms hedging the production risk they face when choosing capital under uncertainty about the future demand shock. Therefore their entry in the permit market results in higher aggregate capital and output by Result 2. Moreover the reward for holding permit inventories, i.e. expected returns, is reduced since the risk is transferred from the firm side to the speculator side: when speculators are active in the permits market the compensation for holding a single permit is indeed reduced by Result 1. Finally the improvement in risk-sharing motives reduces price volatility as well.

The main driver of the above results is the change in the aggregate risk-bearing capacity following (risk-averse) speculators' entry in the market. In order to assess the impact of risk-

aversion on our results we have explored the case of risk-neutral speculators as well. Risk-neutrality seems a plausible assumption when speculators are relatively large players able to diversify the final output demand uncertainty across several financial instruments. Risk-neutral speculators allow to pin down the first period permit price to its actuarially fair value, i.e.  $p_1 = E(p_2)$ . If this condition is satisfied, then speculators are ready to accommodate firms' inventory imbalance. This means that in equilibrium there is no premium for holding permits, i.e.  $E(R) = 0$ , which clearly stems from the fact that profit volatility does not affect speculators expected utility under risk-aversion. It can be shown that in this case the date 1 equilibrium permit price is

$$p_1^{RN} = (1 - \alpha) \left[ \left( \frac{\alpha}{r} \right)^\alpha \mu \right]^{\frac{1}{1-\alpha}}. \quad (27)$$

At  $p_1^{RN}$  speculators buy permits from firms along the same lines of Proposition 8. As is clear from eq. (27), only changes to the expected demand shock affect  $p_1^{RN}$  since  $\mu$  captures the expected profitability of holding permits –while the initial supply of permits  $S_0$  and the volatility of the demand shock  $\sigma^2$  do not enter eq. (27). Moreover, aggregate capital becomes linear in  $S_0$  according to  $K^{RN} = \left( \frac{\alpha\mu}{r} \right)^{\frac{1}{1-\alpha}} S_0$  and return volatility depends positively on the demand shock uncertainty  $\sigma^2$  and negatively on  $\mu$ , but is not affected by the amount of permits issued  $S_0$ . This again is a consequence of the actuarially fair permit price at date 1 in eq. (27).

### 2.3 Optimal environmental policy

The agency objective is defined as in subsection 1.3 thus resulting in the optimization problem (22) where the constraints are now given by eqs. (11), (15') and  $S_0 \geq 0$ . The existence and uniqueness of a solution to the agency problem are still guaranteed by Proposition 5 (intuitively this is because the threshold  $\hat{\delta}$  for the marginal damage in eq. (23) does not depend on the market risk-bearing capacity). Similarly, Proposition 6 continues to hold in the presence of speculators.

We now turn to evaluate how speculators affect the optimal amount of permits issued by the agency and the associated aggregate welfare. Suppose firms only are allowed to trade in

the permits market, as in the baseline model of section 1. Provided marginal damage costs are sufficiently low, the agency optimally sets the number of permits to  $\hat{S}_0$  thus achieving the aggregate welfare  $W(\hat{S}_0)$  (see Proposition 5). How does the agency react if the market opens to speculators as well? As previously noted, the entry of speculators increases the market risk-bearing capacity,  $\tau' > \tau$ . It turns out that what matters for the socially optimal level of pollution –as well as associated welfare– is the relationship between the agency and the market risk-bearing capacity, i.e. between  $\tau_A$  and  $\tau'$  :

**Proposition 7** *Let  $\delta < \hat{\delta}$ . Then*

$$\begin{aligned} \frac{d\hat{S}_0}{d\tau'} &\geq 0 \quad \text{and} \quad \frac{dW(\hat{S}_0)}{d\tau'} \geq 0 \quad \text{if } \tau_A \geq \tau'. \\ \frac{d\hat{S}_0}{d\tau'} &< 0 \quad \text{and} \quad \frac{dW(\hat{S}_0)}{d\tau'} < 0 \quad \text{if } \tau_A < \tau'. \end{aligned}$$

According to Proposition 7 pollution and social welfare exhibit a non-monotonic relationship with the risk-bearing capacity of the market. In fact, both  $\hat{S}_0$  and  $W(\hat{S}_0)$  increase in the market risk-tolerance when  $\tau'$  lies between  $\tau$  and  $\tau_A$ , reach a maximum at  $\tau' = \tau_A$ , and decrease<sup>15</sup> for  $\tau' > \tau_A$ . The intuition underlying such dynamics goes as follows. Note from eqs. (21) and (22) that the agency objective function can be equivalently rewritten as

$$W(S_0) = \Pi_A(S_0) - \delta S_0, \tag{28}$$

with

$$\Pi_A(S_0) = [E(Y) - rK] - (2\tau_A)^{-1} V(Y), \tag{29}$$

where both  $K$  and  $Y$  are functions of  $S_0$ . Expression (28) clarifies that there are two factors affecting aggregate welfare. The first is the social utility of production,  $\Pi_A(S_0)$ , which itself comprises two terms by eq. (29): the expected aggregate production net of firms' consumption of capital,  $E(Y) - rK$ , and aggregate production variability as perceived by the (risk-averse) environmental agency,  $(2\tau_A)^{-1} V(Y)$ . Thus  $\Pi_A(S_0)$  is the risk-adjusted

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<sup>15</sup>If  $\tau_A < \tau$ , then both  $\hat{S}_0$  and  $W(\hat{S}_0)$  are monotonically decreasing in  $\tau$ .

social utility of production. Firms' activity also brings social disutility (pollution) which accrues to the second term in eq. (28),  $\delta S_0$ . If marginal damage costs are sufficiently low, then by Proposition 5 the agency sets the initial amount of permits to the social optimum  $\hat{S}_0$  equating marginal utility of production to marginal disutility of pollution, i.e.

$$\frac{\partial}{\partial S_0} \Pi_A (\hat{S}_0) = \delta. \quad (30)$$

Consider an economy where the agency's risk tolerance is aligned to the one of the market ( $\tau_A = \tau'$ ). In this case one can show that the optimal amount of permits coincides with the solution of a social planner problem, say  $\hat{S}_0^*$ , and welfare is maximal. Figure 3 (left panel) graphically represents the optimality condition (30) for  $\tau_A = \tau'$ .

Whenever  $\tau_A \neq \tau'$  firms do not react as expected by the agency. If  $\tau' < \tau_A$ , firms are too conservative from the agency standpoint: aggregate capital, production and a fortiori the risk-adjusted social utility of production are too low. On the contrary, when  $\tau' > \tau_A$  the agency is more risk-averse than the market. In such a case firms are 'too liberal' from the agency's point of view: both  $K$  and  $Y$  are too large and output volatility  $V(Y)$  negatively affects the social utility of production by (29).

Moreover, the change in the market risk-tolerance affects the marginal utility of production  $\frac{\partial \Pi_A}{\partial S_0}$  as well. Starting from the situation  $\tau_A = \tau'$ , either a decrease ( $\Delta\tau' < 0$ ) or an increase ( $\Delta\tau' > 0$ ) in the market risk-tolerance implies that firms do not react any more as expected by the agency, so that  $\frac{\partial}{\partial S_0} \Pi_A (S_0)$  moves inwards as depicted in Figure 3 (right panel). The agency cannot keep the same amount of permits because at  $\hat{S}_0^*$  the equality between social marginal benefits and costs in eq. (30) is violated, i.e.  $\frac{\partial}{\partial S_0} \Pi_A (\hat{S}_0^*) < \delta$ . The agency reacts to the change in the market risk-aversion by reducing the number of permits to  $\hat{S}'_0 < \hat{S}_0^*$ . Note that this reduction further decreases the risk-adjusted social utility of production as measured by the area under the solid curve  $\frac{\partial \Pi_A}{\partial S_0}$  over the interval  $[\hat{S}'_0, \hat{S}_0^*]$ . However this negative effect on social welfare is more than compensated by the simultaneous reduction in pollution as measured by the rectangle of width  $\delta$  and length  $\hat{S}_0^* - \hat{S}'_0$ . Therefore the marginal social utility of production increases from  $\frac{\partial}{\partial S_0} \Pi_A (\hat{S}_0^*) < \delta$  to  $\frac{\partial}{\partial S_0} \Pi_A (\hat{S}'_0) = \delta$

and optimality is restored.

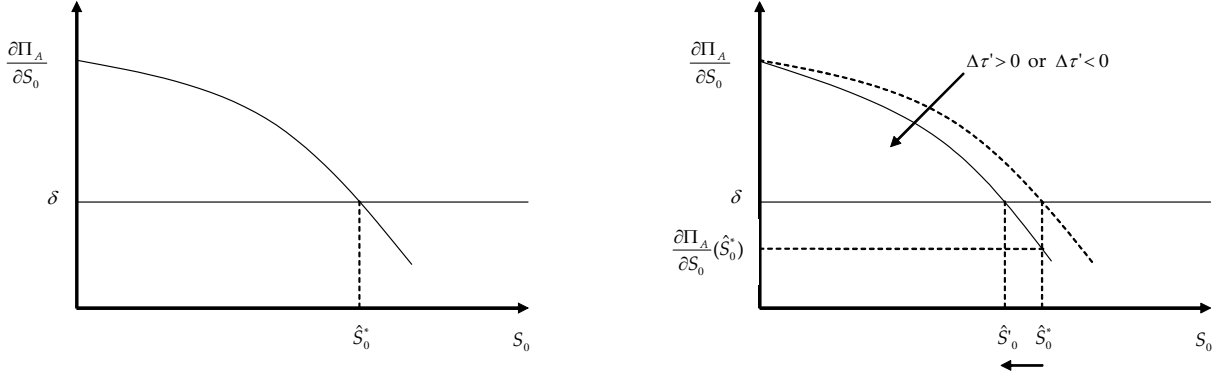


Figure3: Speculation and the optimal amount of permits

The policy implications of our analysis with respect to speculation are therefore as follows. If  $\tau_A > \tau$ , then the regulator should allow speculators on the permit market under the condition that they do not increase the market risk-bearing capacity  $\tau'$  too much (that is, above  $\tau_A$ ). If  $\tau_A < \tau$  then firms alone already make too ‘risky’ decisions from the agency’s perspective, and speculators’ entry should be avoided.

In order to assess the impact of risk-aversion on our predictions we have explored the case of a risk-neutral agency as well. In this situation the production variability does not affect social utility of production, which is simply the expected production net of capital consumption,  $\Pi_A^{RN}(S_0) = E(Y) - rK$ . Even in this case, one can easily check that social utility of production is still concave in the amount of permits, i.e.  $\frac{\partial^2 \Pi_A^{RN}}{(\partial S_0)^2} < 0$ , so that Proposition 5 continues to hold. A risk-neutral agency is characterized by infinite risk-tolerance which means that  $\tau_A > \tau'$  for any finite market risk-bearing capacity. By Proposition 7 this is a situation where firms are always too conservative from the agency standpoint. Any increase in  $\tau'$  –induced for example by the entry of speculators– is then welcomed by the agency by increasing the number of permits and fostering economic activity.

### 3 Discussion

#### 3.1 Results robustness

The sharp policy implications delivered by our model mainly hinges on the comparative statics we perform on several variables of interest, such as permit prices (Propositions 2 and 3) aggregate capital (Proposition 4) and social welfare (Propositions 5, 6 and 7) as well as their counterparts in the presence of speculators. We turn to describe how our predictions are robust to changes in the assumptions we used so far. These changes relate to the normality of the demand shock, the CARA utility function, the Cobb-Douglas production function and the linearity of the damages.

Imposing the demand shock to be normally distributed considerably adds tractability, but is indeed not crucial to get our results. For example, we have explored the case of a binomial demand shock, i.e.<sup>16</sup>

$$\theta = \begin{cases} \mu + \sigma & \text{prob. } 1/2 \\ \mu - \sigma & \text{prob. } 1/2. \end{cases} \quad (31)$$

Under this distributional assumption we have checked that both price uniqueness (Proposition 1) and the comparative statics in Proposition 2 would go through.<sup>17</sup> Finally, there exists a unique solution to the agency optimization problem provided  $\delta$  is low enough along the same lines as Proposition 5.

In order to assess the relevance of the CARA utility function for our results, we have explored the CRRA case in the presence of a binomial demand shock like in (31). Already at the level of the market equilibrium (that is for  $S_0$  given) computations turn quickly to a highly nonlinear system of equations that can only be solved numerically and multiple equilibrium values of the permits price are possible. This loss in tractability has to be expected since it arises even in simpler frameworks than ours (see among others Bernardo and Welch [3]).

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<sup>16</sup>In order to guarantee non-negative output we also impose  $\mu \geq \sigma$ .

<sup>17</sup>In particular one can show that in this case the equilibrium price  $p_1$  is bounded below by  $(1 - \alpha) \left[ \left( \frac{\alpha}{r} \right)^\alpha (\mu - \sigma) \right]^{\frac{1}{1-\alpha}}$  and above by  $(1 - \alpha) \left[ \left( \frac{\alpha}{r} \right)^\alpha \mu \right]^{\frac{1}{1-\alpha}}$ .

However, under CRRA it is easy to see that the initial endowments of permits of the firms will play a role in the determination of the price of permits (as in Baldursson and von der Fehr [2] and contrary to our reference case).

We have checked that the existence of an equilibrium price in the permits market (see Proposition 1) extends to general linear homogenous production functions

$$y_i = \theta k_i f\left(\frac{e_i}{k_i}\right), \quad (32)$$

where  $f(\cdot)$  satisfies standard conditions. One can indeed derive restrictions on the behavior of  $f$  when the energy to capital ratio diverges to  $+\infty$  so that the existence and uniqueness of a positive equilibrium permit price are guaranteed.<sup>18</sup> Under such restrictions on the production function, we were able to derive an upper bound on the marginal damage  $\delta$  such that the agency welfare maximization admits a positive maximum, thus generalizing the inequality (23) in Proposition 5.

Insofar we have assumed that damages are linearly related to pollution. To test whether this restrictive assumption is crucial for our results, we have explored the case where damages are expressed as  $D(S_0) = \frac{\delta}{2}S_0^2$  and social welfare (20) becomes

$$\theta K^\alpha S_0^{1-\alpha} - rK - \frac{\delta}{2}S_0^2. \quad (33)$$

Under quadratic damages, it can be shown that the agency welfare function is continuous, bounded from above and positive on some interval for  $S_0 > 0$ . Therefore, the agency optimization problem admits a maximum like it happens in the linear damages case we have hereby dealt with. However what is new with respect to the case where  $D(S_0) = \delta S_0$  is that the existence of a maximum obtains without imposing any condition on the parameter  $\delta$  like the inequality (23). This arises because marginal damages are now linear homogenous, i.e.  $D'(S_0) = \delta S_0$ . For a given  $\delta$ , allocating a (sufficiently) low amount of permits to firms reduces marginal damages so that the no production –and therefore no pollution– issue does not arise here.

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<sup>18</sup>In the case of CES production function, such condition restricts the elasticity of substitution between the two inputs from below.

In summary, departures from our framework allowing for a binomial demand shock, a general homogeneous production function or a quadratic damage function do not impair equilibrium uniqueness for both the permits market and the agency problem. The impact of speculators on the aggregate welfare however is unclear. However, dropping the CARA in favour of the CRRA utility function has unclear consequences because computations **become cumbersome**.

## 3.2 Policy implications

Three important issues deserve additional comments. The first two relate to our main results, namely the comparative statics for permits prices and returns (Proposition 2 and 3) and social welfare (Proposition 7). The third issue concerns short-selling constraints and the introduction of derivatives.

1. A careful look at existing markets such as the US market for SO<sub>2</sub> permits or the European market for CO<sub>2</sub> permits reveals that permits prices may significantly fluctuate through time. Of course, numerous elements explain these fluctuations, such as changes in oil or electricity prices, weather, political decisions, etc.<sup>19</sup> Our findings suggest that, besides these exogenous shocks, risk aversion and risk hedging behavior may explain part of the price movements observed on markets for emission permits.<sup>20</sup> Indeed, we have shown that (expected) permit prices tend to rise through time (see Proposition 3). This result departs from the existing contributions on transaction costs (see e.g. Stavins [16]) or intermediation (see e.g. Germain et al. [8]) in markets for emission permits, which emphasizes the possible existence of a spread (i.e., a difference between the price at which permits are sold and the price at which permits are purchased). Here, we observe an intertemporal spread, i.e., a situation in which *expected* permit prices change through time. Moreover, we have also shown that the volatility of the

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<sup>19</sup>On this topic, see for instance Albrecht et al. [1] for the US SO<sub>2</sub> market and Convery and Redmond [5] as well as analyses by PointCarbon (see [www.pointcarbon.com](http://www.pointcarbon.com)) for the EU CO<sub>2</sub> market.

<sup>20</sup>Some authors, including Zhao [20], have dealt with the consequences of a change in the permits price. Here, we give a possible explanation for such changes.

prices is influenced by the presence of speculators and, more generally, by the market risk-tolerance. An increase in the risk-bearing capacity tends to stabilize prices, and vice-versa.

2. Classical analyses of the tradable emission permits instrument do not account for the participation of speculators on the market. The main policy implications of our analysis depend on the degree of risk aversion of the agency w.r. to the one of firms and speculators. Since the entry of speculators in the permits market increases the overall risk-bearing capacity, then by Proposition 7 institutional rules should favor the presence of speculators on markets for emission permits (by contrast with the situation where only polluting firms are present on the market). However, allowing speculators to operate on the emission permits market should be granted only to a certain extent, i.e. only as long as the agency's risk tolerance remains greater than the market risk-bearing capacity. As previously noted, it is quite difficult to measure corporate risk attitudes from an empirical standpoint and therefore to assess whether the risk tolerance of firms is larger or smaller than the agency's. One could argue that regulators active on several markets have more possibilities than firms to diversify demand risk in a single market, thus leading to larger risk tolerance for the environmental agency than for firms. In this case opening the permits market to speculators is welfare improving as long as their entry doesn't raise the overall market risk tolerance too much. In the limiting case where the agency is risk-neutral, speculators always foster economic activity and welfare since the agency is concerned with the level of production and not its volatility.
3. Another policy-oriented issue is the extent to which short-selling of emission permits is likely to take place. As shown in Proposition 3, firms find it optimal to unload some or the whole stock of permits to speculators during the first trading round. Once uncertainty is resolved, i.e. during the second trading period, firms buy back permits and use them in the production process. The (expected) price differential compensates speculators for bearing the risk of holding permits during the first period. In some situations, typically when their degree of risk-tolerance is high, speculators might be

willing to hold more than the total amount of permits available in the economy. One may therefore wonder if the total amount of permits purchased by speculators should not be constrained by the total amount of permits allocated by the agency (i.e.,  $x_1 \leq S_0/n_S$ ). Exactly as on financial markets, such a constraint is not likely to be relevant. In fact, institutions can be designed in order to allow the agents to hold (temporarily) a negative amount of emission permits in their account, i.e., to allow for short selling.<sup>21</sup> Otherwise, the introduction of forward contracts (with cash delivery) may play the same role in ruling out short-selling constraints. A long (resp. short) forward contract specifies the future date and the price at which one agent will sell (resp. buy), say, one permit. The introduction of such contracts in our model is straightforward. The first period is devoted to the forward trading (instead of being a spot market):  $x_{1j}$  is the number of forwards sold by speculator  $j$  (in period 1) and  $x_{2j} = -x_{1j}$  is the delivery of the permits (in period 2); finally,  $p_2$  is the price of the permits on the spot market (in period 2). If contracts allow for cash delivery (as opposed to physical delivery, i.e., in terms of emission permits), speculators may purchase an amount of permits that is larger than the amount of permits available in the economy. Thus a situation with forward contracts allowing for cash delivery is isomorphic to the analysis we have so far conducted.

## 4 Conclusion

We have analyzed the impact of speculators on the emission permits market when agents are risk-averse. Our main results are the following. First, permits price are expected to increase through time, so that there is some room for risk hedging by firms. Second, an increase in the risk-bearing capacity of the market (due for instance to an increase in the measure of speculators or a decrease in the risk aversion of firms or speculators) rises expected social welfare and pollution up to a certain point depending on the agency's risk tolerance.

The setup we have developed relies on several specific assumptions that allow to obtain

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<sup>21</sup>For instance, on the recently launched European market for CO<sub>2</sub> permits, shortselling is possible.

clear cut predictions. Some of these assumptions (normality of risk, Cobb-Douglas production function, linear damages) do not appear to be crucial for most of our results. However, extending our analysis to alternative utility functions such as CRRA is a much more difficult task. This issue certainly deserves further investigation in order to assess the robustness of our results.

Our framework lends itself to other extensions and further investigation. First, the determination of an endogenous measure of speculators in the market would be of interest. A second extension would be allowing speculators to invest in assets other than emission permits. This way one can address the effect of portfolio diversification on our results. Portfolio choices would be driven by the comovement of permits with other available assets so that to speculators the risk of permits would be measured by a beta-like coefficient rather than demand shock volatility as it is in our current setup. Thirdly, to understand more comprehensively the role of speculators on markets for emission permits, one may introduce feedback (noise) traders à la De Long et al. [7]. These authors focus on speculators who make their trading decision by simply extrapolating past prices. They show that the presence of such traders may cause the emergence of a bubble. A fourth interesting extension would be to introduce informational asymmetries between the different types of agents. For instance, one could consider that, by performing market analyses, speculators are better informed than firms about possible macroeconomic shocks.

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