Optimism, Pessimism and Financial Markets

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Abstract

We analyze a model where unrealistic and realistic informed traders trade a risky asset with a competitive market maker. Unrealistic traders misperceive the expected returns of the risky asset, the volatility of the asset returns and the variance of the noise in their private signal. We show that unrealistic traders may earn higher expected profit than realistic traders and that the existence of equilibrium depends crucially on the amount of over-trading on private information. We then allow the market maker to be unrealistic. An optimistic (pessimistic) market maker exacerbates (alleviates) market breakdown. Due to her unrealism a price bubble might occur.

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1 Introduction

Economic and financial theory have widely used the assumption that agents behave rationally. Such an assumption has led to the failure of explaining some properties observed in financial markets like (*i*) the low responsiveness or sometimes high responsiveness of the price to new information [Ritter (1991) and Womack (1996)], (*ii*) the excessive volume traded [Dow and Gorton (1997)], (*iii*) underreaction or overreaction of market participants [Debondt and Thaler (1985)], and (*iv*) the excessive volatility observed in financial markets [Shiller (1981, 1989)]. In order to explain these properties, financial economists have departed from the rationality assumption and have instead assumed that investors may have some psychological traits which would lead them to behave irrationally.

There is a large body of evidence in the psychology literature that people, with good mental health, do not have accurate perception of themselves and their surrounding world. People's perceptions have a tendency to be positively biased, i.e. people hold "positive illusions". "Positive illusions" have been widely documented in psychology [Taylor and Brown (1988, 1994), Langer (1975) and Weinstein (1980), to name but a few]. Taylor and Brown (1988, 1994) analyze the "better than average" effect, Langer (1975) focuses on the illusion of control i.e. the individuals' tendency to overestimate the control they have over outcomes, whereas Weinstein (1980) looks at unrealistic optimism. Unrealistic optimism is defined as the people's tendency to systematically overestimate the probability that good things will happen to them and, at the same time, to underestimate the probability that bad things will happen.[1] [2]

Many examples of the optimism bias can be found. It has been observed that door-to-door representants attributing failure to outside factors are more likely to be successful. This optimism bias can lead to superior performances.[3] Sutherland (1992) finds that 95% of the British drivers believe that they are above average. Langer (1975) documents the illusion of control. The experiment is as follows: participants are either given a lottery ticket or asked to draw one. Participants were then asked the selling price for the lottery ticket. Participants who had drawn themselves the ticket would ask twice as much as participants who had received the ticket. This higher price presumably reflected the belief of a higher chance of winning.

Financial praticitioners are well aware of the existence of such a trait in markets. Indeed the terminology used in order to qualify the market proves that fact: they interchangeably use optimistic market for bullish market and pessimistic market for bearish market. Some financial institutions have tried

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[1] Harris and Middleton (1994), and Hoorens (2001) look at the "comparative optimism". These two papers focus on the illusion of control and optimism about health: people think on being less at risk about their health but no more in control than others.

[2] One way psychologists test the presence of that trait is to ask their subjects how they behave in bad times. If in bad times they expect the best, they can be considered as optimistic.

[3] Following that observation, Seligman designed a test, the Seligman Attributional Style Questionnaire (SASQ), that would enable companies to select candidates, when hiring, displaying that optimistic bias.

to quantify it. In October 1996, the Union des Banques Suisses together with Gallup Organization have launched the *Index of Investor Optimism.* This index measures the level of optimism in the American market. The Index had a baseline of 100 when it was first established in October 1996. Just before the internet bubble crash in January 2000, that Index attained a score of 178. It got its lowest score of 5 in March 2003, however it stands at 80 in February 2006.[4] This variation of the value of the Index shows the presence of relative period of optimism and pessimism in the American market. We are also aware of some portfolio managers who believe that their stock selection is the best one. More generally this optimism/pessimism trait is likely to be present for most people. Typical examples of periods of optimism or frenzies followed by periods of pessimism in financial markets are the existence of bubbles and crash. Famous bubbles include the Dutch Tulip Mania of the 1630's, the Mississippi Bubble and the South Sea Bubble of the 1720's, and the Internet Bubble of the 1990's.[5] Shiller (2000) provides examples of optimistic behaviors.

Although the existence of such a trait is established, it has received little attention by financial economists. Our aim is to fill up that gap. Indeed, our study follows directly the idea that some investors display a psychological trait such as "positive illusions/negative illusions". We call the former (latter) investors optimists (pessimists). We model this trait as being made up by two independent parts: (*i*) pure optimism/pessimism which represents the misperception of prior information (expectation and variance) and, (*ii*) pure overconfidence/underconfidence which is the misperception of the precision of private information. Few recent papers analyze this trait, however all these studies do not look at the impact of the presence of unrealistic traders in financial markets. Most of these studies are based on models issued from the corporate finance literature. Bénabou and Tirole (2002) look at the value of self-confidence for rational agents and at their behavior used to enhance it. An individual has to decide whether or not to engage in a task. The long term payoff of that task depends on his imperfectly known ability and on sure short term cost which is known to him. They show that the task is undertaken only if the agent has sufficient self-confidence in his talent. Van Den Steen (2004) shows that rational agents with differing priors tend to be overoptimistic about their chances of success. In that setting, he also proves that, as suggested by the door-to-door salesmen example above, agents tend to attribute success to be their own whereas failure to exogenous factors. In Brocas and Carrillo (2004), optimism about the chance of success of a project may lead entrepreneurs to invest in that project without gathering further information. This entrepreneurial optimism leads to excessive investment. De Mezza and Southey (1996) find that entrepreneurs self select from the part of the population displaying an optimistic bias. Manove and Padilla (1999) look at the screening problem faced by bankers in order to separate optimistic entrepreneurs from realistic

[4] A detailed methodology used to compute that index can be found at www.ubs.com/investoroptimism. The monthly level of the index is also given from its launch date up to now.

[5] The first three bubbles are well documented in Garber (2000).

ones. Optimistic entrepreneurs have perceptions biased by wishful thinking. They show that, because of the existence of optimistic entrepreneurs, competitive banks may not be sufficiently conservative in their lendings. In Manove (2000), some entrepreneurs are unrealistically optimistic about their firms productivity. He shows that unrealistic entrepreneurs may earn more than realistic entrepreneurs and may even drive out of business all realistic entrepreneurs. This result is echoed by Heifetz and Spiegel (2004) in a different framework. In a game where agents meet per pair and then interact, they show that agents displaying optimism or pessimism will not disappear in the long run but will takeover the entire population. In their paper, optimistic (pessimistic) agents over- (under-) estimate the impact of their actions. Our study differ from the previous studies on optimism as none of the above works look at the impact on financial markets of having optimistic (pessimistic) traders. Cornelli et al. (2005) is the only other work, we are aware of, looking at the possibility, for a subset of investors, to be optimistic or pessimistic. They establish the existence of unrealistic traders possibly optimistic or pessimistic in the grey market (pre-IPO market). In our paper, we assume the existence of a subset of informed investors being optimistic and/or pessimistic and derive the equilibrium and its properties under that assumption.

The following paper is an attempt to answer some questions concerning the performance of a financial market when both unrealistic and realistic traders are present. What is the trading behavior of unrealistic traders and their impact on prices and price efficiency? Can realistic traders counteract the effect of the unrealistic traders' trading behavior on the price, or do they exacerbate the effect of the unrealistic traders? In other words, what is the result of the strategic interaction of the different types of traders (realistic and unrealistic) onto the price and price efficiency? Does one group outperform the other? And finally, what is the effect on the price and trading behavior of both types of investors if the price setter is himself/herself unrealistic?

The introduction of a psychological bias or trait at the investors' level is not new in finance, a proof being the abundant literature assuming that investors are overconfident, i.e. that investors believe that their private information is more precise than it actually is.[6] Hilton et al. (2004) show that overconfidence or "miscalibration" and "positive illusions" differ even if those two traits share some characteristics (in both cases the agent over-values his/her ability). In order to model the overconfident behavior, it is assumed that investors overestimate the precision of their private information. Most of that literature predicts that overconfident investors trade to their disadvantage. In other words, overconfident investors get lower expected profit than their rational counterpart [Odean (1998b), Gervais and Odean (1999), Caballé and Sákovics (2003), Biais et al. (2004) among others]. However, Kyle and Wang (1997) and Benos (1998) find that overconfident traders may earn larger expected profit than rational ones. Moreover, a common finding to all these papers except Caballé and Sákovics (2003) is that trading volume, price volatility as well as price efficiency

[6] See Odean (1998).

increase with the level of overconfidence.

We study a more general model of optimism and pessimism. In our article we analyze a financial market where a market maker and several traders exchange a risky asset normally distributed with mean 0 and variance σ_v^2. That model is based on the seminal paper by Kyle (1985) and on Admati and Pfleiderer (1988) and Didri and Germain (2004) which extend the previous model to the case of imperfect competition for the informed traders. In such a framework, unrealism can be modelled in two different ways. First, unrealistic traders are unrealistic about the returns of the risky asset (mean of prior information) and therefore believe that the expectation is a rather than zero. An optimist (pessimist) believe that a is positive (negative), as a increases in absolute terms the trader is more optimistic/pessimistic. This is studied in a first setting. Indeed, it seems intuitive to think of an optimistic (pessimistic) trader as a trader believing that the expected returns of the risky asset are higher (lower) than they actually are. We dedicate a section of the paper to study that case as it is not present in the literature. This model is one of the most simple in order to study optimism/pessimism. Second, we turn to a more general setting where unrealistic traders are unrealistic about the returns of the risky asset, as in the first setting, and about the volatility of the returns of the asset as well as the variance of the noise in his private information (over- or under-confidence). In this second setting, we model the misperceptions of the variances as two parameters: κ_1 (optimistic/pessimistic component) and κ_2 (overconfident/underconfident component) which altogether characterize the degree of misperception of the two variances. The smaller the κ parameter, the more unrealistic is the trader. An optimistic (pessimistic) trader under-scales (over-scales) the volatility of the risky asset σ_v^2 by a parameter κ_1 and under-scales (over-scales) the precision of his own signal σ_ε^2 by a parameter κ_2.[7] In such a model, an optimistic (pessimistic) trader thinks that prior information is more concentrated (diffused) around the misperceived mean, a. Probabilities that good things can happen are then higher for optimistic traders, moreover optimistic traders are less sensitive to risk as they underestimate the market risk. In both settings, traders are strategic, i.e., when computing their order they take into account the impact of the order onto the price.

Our results are the following. In the first setting (misperception of the returns only), since the misperception of the mean has an effect on the market order unrealistic traders submit, we find that optimistic (pessimistic) traders trade larger quantities when purchasing (selling) or trade smaller quantities when selling (purchasing). Even though, the aggregate order flow faced by the market maker is different from the one in the "all realistic case" (where all the traders are realistic), the price and the market depth are equal for the two cases.[8] Finally, the expected profits of the realistic and the unrealistic traders

[7] In Odean (1998b) those two parameters define the level of overconfidence. Nevertheless, the author does not study the impact of the variations of those parameters on the traders and Odean (1998b) assumes that κ_1 is greater than one whereas we assume that for optimistic traders it is lower than one.

[8] The price and market depth are equal to the one predicted by a standard model à la

are identical. In that market it is not costly to be unrealistic.
In the second setting (misperception of the returns and of the two variances σ_v^2 and σ_ε^2), we look at two different situations: (i) a realistic market maker trades with realistic traders and with only one type of unrealistic traders, (ii) the market maker is herself optimistic or pessimistic.[9] In both instances, (i) and (ii), we find that, compared to a situation where all traders are realistic, realistic traders reduce (increase) their trading intensity when unrealistic traders over- (under-) trade. However, we show that, for certain market conditions, there exists a situation where both types of traders over-trade. This is the case when the misperception of the volatility is smaller, however not to small, than the misperception of the noise in the signal ($\kappa_1 > \kappa_2$). This trading behavior has an effect on market depth which can be higher or lower than in the "all realistic case". Importantly, we show that, for both a realistic and an unrealistic market maker a market breakdown occurs if both types of traders over-trade excessively. However, a market breakdown is more likely to occur with an optimistic market maker than with a pessimistic one. Indeed, an optimistic (pessimistic) market maker sets a higher (smaller) market depth than a realistic one. As a response to the increased (decreased) market depth , both types of traders increase (decrease) their trading intensity. As they increase (decrease) their trading intensity on private information, a market breakdown is more (less) likely to occur. Due to the market maker's unrealism an irrational price bubble might occur.[10] We finally study, the traders' expected profit and find, in both instances (i) and (ii), that unrealistic traders can earn on average more or less than their realistic counterpart. Unrealistic traders can also obtain negative expected profits. We also show that an unrealistic market maker can obtain non-zero expected profit.

In other models like Benos (1998) or Daniel, Hirshleifer and Subrahmanyam (2000) overconfidence is defined by κ_2, only. By allowing κ_1 and κ_2 parameters to characterize the unrealistic behavior, our findings are more general than the ones obtained in Benos (1998), Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (2000) to name but a few.[11] The effect of the parameter defining the over- or under-confidence is known, however the combination of the two misperceptions of the variances is not present when traders are only overconfident. Some of our results are qualitatively and quantitatively different from the ones obtained with overconfident traders. Indeed, the price function

Admati and Pfleiderer (1988). Indeed, the quantity of information conveyed by the market orders is the same as in Admati and Pfleiderer (1988).

[9] The market maker's unrealism is characterized by the misperception of the returns of the asset and the misperception of its volatility. Given that the market maker does not receive any private information, she does not display any overconfidence or underconfidence trait. Assuming that the market maker is unrealistic enables us to also study the impact of unrealistic liquidity suppliers in a standard demand and supply framework.

[10] We characterize an irrational bubble as the increase of the level of prices (on average). Indeed when market makers are optimistic they set a level of prices which is above the one predicted by a model à la Admati and Pfleiderer (1988) with realistic agents only.

[11] All these authors implicitly assume that $\kappa_1 = 1$. Benos (1998) studies the extreme case where overconfident traders perceive their private information as being non noisy, that is $\kappa_1 = 1$ and $\kappa_2 = 0$.

displays properties different than the ones obtained in the overconfident case. The presence of unrealistic traders has an effect on both the intercept and the slope of the price function. The first effect on the intercept is not present in any of the studies with overconfident traders. The second effect on the slope is present with overconfident traders, however we find that market liquidity, depending on the values of the parameters, can be greater or smaller than the market liquidity with realistic traders only which is not the case in most papers on overconfidence where liquidity increases with the level of overconfidence in the market.[12]

The paper by Kyle and Wang (1997) deserves a bit more of attention as it is very close to our work. They explore the strategic interaction between an unrealistic and a realistic trader. The unrealistic trader is defined as a trader misperceiving the variance of the noise of his/her own signal and having some beliefs concerning the variance of the noise of the other trader's signal. This trader can either be overconfident or underconfident. They obtain that the equilibrium only depends on the trader's misperception of the variance of the noise of his/her own signal. They find that when an unrealistic trader is able to credibly commit to trade a larger quantity, this trader can outperform the realistic trader. They show that an overconfident trader is able to commit whereas an underconfident is not. We do obtain the same result for an optimistic trader. However, some of our results concerning pessimistic traders are qualitatively different. In our model, a pessimistic trader believes that prior information has a greater variance. This enables him/her to commit to trade larger quantity. As a consequence, we find that pessimistic trader can outperform realistic traders.

The paper unfolds as follows. In the next section, the general model is presented along with the definition of an equilibrium for our model. In section 3, the model is solved for the additive misperception (misperception of the returns) alone as well as for the case where the additive and the multiplicative (misperception of the variances) misperceptions are combined for the case when the market maker is realistic. In section 4, we derive the equilibrium under the assumption that the market maker is herself optimistic or pessimistic and trades with only one type of unrealistic traders. The last section summarizes our results and concludes. All proofs are gathered in the appendix.

2 Model

We study a financial market where a market maker and several traders exchange a risky asset whose future value $\tilde{v}$ follows a gaussian distribution with zero mean and variance σ_v^2.[13] Traders participating in that market can either be informed or uninformed. The uninformed traders are the so-called noise traders and submit a market order which is the realization of a normally distributed random variable $\widetilde{u}$ with zero mean and variance σ_u^2. The informed traders are

[12] Benos (1998) finds that the market liquidity is increased if overconfident traders are present.

[13] This is also called prior information distribution.

risk neutral and can be one of two types: realistic or unrealistic. N traders are realistic whereas M are unrealistic. Both types of traders have access to private information, i.e. they observe a noisy signal of the future value of the risky asset

$$\tilde{s}_k = \tilde{v} + \tilde{\varepsilon}_k, \text{ with } \tilde{\varepsilon}_k \rightarrow N\left(0, \sigma_\varepsilon^2\right) \quad \forall k = 1, ..., N, N+1, ..., N+M.$$

These two types of traders differ in the beliefs they have about the expectation of the risky asset value (expectation of prior information). Realistic traders correctly believe that the expectation is zero. However, unrealistic traders believe that it is a. That expectation is positive (negative) if they are optimistic (pessimistic). The term unrealistic trader refers to a trader who uses the wrong distribution for the asset return. However, that trader rationally anticipates the behavior of both the market maker and the remaining informed traders.

The strategy of each realistic trader i is a Lesbegue measurable function, $X_i : \Re \rightarrow \Re$, such that $\tilde{x}_i = X_i\left(\tilde{s}_i\right)$ for $i = 1, ..., N$. The strategy of the unrealistic is identically defined: $X_j : \Re \rightarrow \Re$ such that $\tilde{x}_j = X_j\left(\tilde{s}_j\right)$ for $j = 1, ..., M$.

Finally, the market maker is risk neutral and behaves competitively. She observes the aggregate order flow $\tilde{y} = \sum_{i=1}^{N} \tilde{x}_i + \sum_{j=1}^{M} \tilde{x}_j + \tilde{u}$ before setting the price $\tilde{p}$. Let $P : \Re \rightarrow \Re$ denotes a measurable function such that $\tilde{p} = P\left(\tilde{y}\right)$.

The trading protocol is identical to Kyle (1985).

We now give the definition of an equilibrium for our model.

Definition $(X_1^r, ..., X_N^r, X_1^{un}, ..., X_M^{un}, P) \in L^{N+M+1}$ *is an equilibrium if the price set by the market marker is such that*

$$\tilde{p} = E\left[\tilde{v}|\, \tilde{y}\right],$$

and, given that price, the market orders maximize the traders' expected profit conditional on the information received

$$X_i^r \in \arg\max_{x_i^r \in \Re} E\left[\left(\tilde{v} - P\left(\tilde{y}\right)\right) x_i^r \middle|\, s = s_i\right] \quad \forall i = 1, ..., N,$$

and

$$X_j^{un} \in \arg\max_{x_j^{un} \in \Re} E_{un}\left[\left(\tilde{v} - P\left(\tilde{y}\right)\right) x_j^{un} \middle|\, s = s_j\right] \quad \forall j = 1, ..., M.$$

The operator E_{un} denotes the fact that the expectation for unrealistic traders, is computed given their beliefs about the risky asset expectation.

It should be pointed out that all traders know their types. Moreover, all agents know the number of realistic and unrealistic traders as well as if unrealistic traders are optimistic or pessimistic. Traders behave strategically meaning that they take into account the impact of their orders onto the price.

First, in section 3, we solve the model for the case where the market maker is realistic, i.e. she uses the correct distribution of the asset. Second, in section 4, we look at the case where she is either optimistic or pessimistic.

3 Realistic Market Makers

3.1 Additive Misperception

In that subsection we derive the equilibrium where the unrealistic traders have incorrect beliefs about the expectation of the risky asset value. Let us define $\tau = \frac{\sigma_\varepsilon^2}{\sigma_v^2}$.

Proposition 1 *There exists a unique equilibrium of the following form*

$$\begin{aligned} x_i^r &= \beta^r s_i, \quad \forall i = 1, ..., N, \\ x_j^{un} &= \alpha^{un} + \beta^{un} s_j, \quad \forall j = 1, ..., M, \end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u \right).$$

The coefficients for the market orders are such that

$$\begin{aligned} \alpha^{un} &= \frac{2a\sigma_u \tau}{\sigma_v \sqrt{(M+N)(1+\tau)}}, \\ \beta^{un} &= \beta^r = \frac{\sigma_u}{\sigma_v \sqrt{(M+N)(1+\tau)}}. \end{aligned}$$

The price schedule is given by

$$\begin{aligned} \mu &= -a \frac{2\tau}{2\tau + M + N + 1}, \\ \lambda &= \frac{\sigma_v \sqrt{(M+N)(1+\tau)}}{\sigma_u (2\tau + M + N + 1)}. \end{aligned}$$

Given the above, the aggregate order flow faced by the market maker can be written as

$$y = u + \sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} = u + \beta^r \sum_{k=1}^{M+N} s_k + M\alpha^{un} = y^* + M\alpha^{un}.$$

Proof. See Appendix. ■

In the following discussion we only look at the case where optimistic traders are present as the pessimistic case is symmetric.

Both the realistic and the optimistic traders trade with the same intensity on private information ($\beta^{un} = \beta^r$). An optimist adds up a positive part to his market order. In fact since $\alpha^{un} > 0$, if he received a positive signal, he increases his market buy order whereas if he received a negative signal he reduces his market sell order. This change in his demand is proportional to his level of optimism, a. A price bubble does not occur as the market maker is realistic. The price and the market depth (the slope of the price function) are equal to the price and market depth predicted by a model à la Admati and Pfleiderer (1988) with $M+N$ realistic traders. This can be explained as follows. Since unrealistic traders are optimistic and therefore $M\alpha^{un} > 0$, when pricing the risky asset, the market maker faces a larger positive aggregate order flow or a smaller negative one (when selling). Given the additive and deterministic nature of this extra order flow, $M\alpha^{un}$, and given that the market maker rationally anticipates the behavior of the optimists, she can compute its exact size. In addition, this part is independent of $\tilde{v}$, therefore when trying to extract information from the order flow, the market maker uses a "discounted" order flow, called y^*. Since (i) both types of traders respond identically to private information and (ii) their responsiveness to private information correspond to the one obtained in Admati and Pfleiderer (1988), the market depth is also identical to the one predicted by a model à la Admati and Pfleiderer (1988). This results in a downward shift of the price function such that, given an order flow y^* the price is identical to price with an order flow y. Indeed, for a given realization of $\widetilde{u}$ and of $\tilde{s}_k$ $\forall k = 1, ..., N, N+1, ..., N+M$, both observed aggregate order flows, y and y^*, incorporate the same information. This point is illustrated below in figure 1 as well as the case for which pessimists are present. As this is rationally anticipated by the realistic trader, the misperception of the asset return only affects the optimist's market order, i.e. $\alpha^r = 0$.

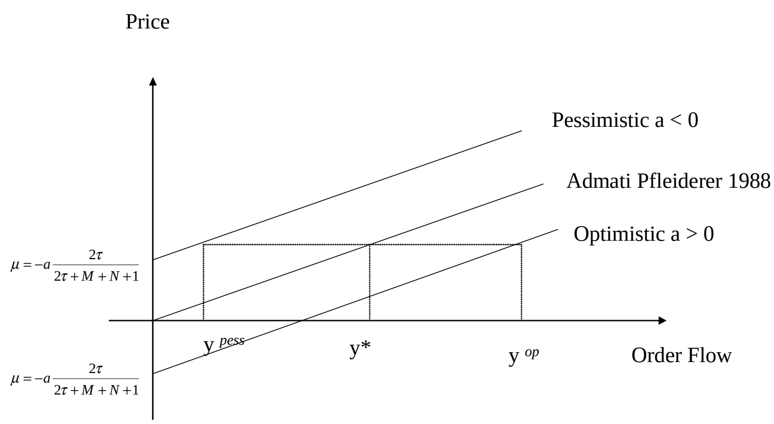

Figure 1: Overall level of prices for the cases where unrealistic traders are pessimistic, no unrealistic traders are present (Kyle (1985)), and unrealistic traders are optimistic.

We now have a look at some of the comparative statics of the model. Except for the overall effect of σ_ε^2 on the market order from the unrealistic, all the comparative statics concerning the market depth, β^r and β^{un} accord to intuition and are similar to Admati and Pfleiderer (1988). We now discuss in more details the effect of σ_ε^2. Let us rewrite the market order from the unrealistic j as follows

$$x^{un} = \beta^{un} (s_j + 2\tau a) .$$

On the one hand, an increase of the noise in the private signal decreases the size of the market order. On the other hand, it increases the weight within the order due to the misperception of the expectation of prior information. In other words, the noisier the information is, the more unrealistic the unrealistic trader is and the opposite is also true. When $\sigma_\varepsilon^2 = 0$, the unrealistic trader trades only on private information, however when $\sigma_\varepsilon^2 = +\infty$ the signal is not informative and the unrealistic trader trades only on his misperception a.

We now look at the unconditional expected profit of the traders.

Proposition 2 *The unconditional expected profit for both types of traders are identical. They are equal to*

$$E(\Pi^{un}) = E(\Pi^r) = \frac{\sigma_v \sigma_u}{2\tau + M + N + 1} \sqrt{\frac{\tau + 1}{M + N}}.$$

Proof. See Appendix. ■

The wrong beliefs about the mean of the returns of the risky asset have no impact on the level of unconditional expected profit achieved by the unrealistic traders. This is due to the fact that the wrong beliefs have only an impact on the overall level of prices and no impact on the liquidity parameter or on the traders' intensity concerning their private information. Again, the comparative statics are identical to the ones obtained in a model à la Admati and Pfleiderer (1988). Both types of traders obtain positive expected profit. In that case, it is not costly to be unrealistic.

3.2 Additive and Multiplicative Misperception

In that subsection, we look at the general case of positive/negative illusions which we call optimism/pessimism. We define it as being made up by two independent parts[14]

- pure optimism/pessimism: misperceptions of the distribution of prior information (expectation and variance),
- pure overconfidence/underconfidence: misperception of the variance of the noise in the private signal.

An optimistic (pessimistic) trader underestimates (overestimates) the variance of both the returns of the asset and the noise in the signal received. An unrealistic trader behaves as if his signal, $\tilde{s}_j = \tilde{v} + \tilde{\varepsilon}_j$ for $j = 1, ..., M$, were drawn according to the following two distributions

$$\begin{aligned} \tilde{v} &\rightarrow N\left(a, \kappa_1 \sigma_v^2\right), \\ \tilde{\varepsilon}_j &\rightarrow N\left(0, \kappa_2 \sigma_\varepsilon^2\right), \end{aligned}$$

[14] This follows the definition given in Hilton et al. (2004).

where $0 < \kappa_1 < 1$, $0 < \kappa_2 < 1$ for optimistic traders and $\kappa_1 > 1$, $\kappa_2 > 1$ for pessimistic traders. Whenever $\kappa_1 = \kappa_2 = 1$, the unrealistic trader does not misperceive both variances.

We now characterize the equilibrium.[15]

Proposition 3 *Whenever*

$$M\kappa_1 (1+2\tau)^2 [\kappa_1 (1-\tau) + 2\kappa_2 \tau] + N (\kappa_1 + 2\kappa_2 \tau)^2 (1+\tau) \geq 0. \qquad (1)$$

there exists a unique equilibrium of the following form:

$$\begin{aligned} x_i^r &= \beta^r s_i, \quad \forall i = 1, ..., N, \\ x_j^{un} &= \alpha^{un} + \beta^{un} s_j, \quad \forall j = 1, ..., M, \end{aligned}$$

$$p = \mu + \lambda y = \mu + \lambda \left(\sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u \right).$$

The coefficients are such that:
for the optimistic/pessimistic traders

$$\begin{aligned} \alpha^{un} &= \frac{2(1+2\tau)\kappa_2 \tau \sigma_u}{\sigma_v \sqrt{M\kappa_1 (1+2\tau)^2 [\kappa_1 (1-\tau) + 2\kappa_2 \tau] + N (\kappa_1 + 2\kappa_2 \tau)^2 (1+\tau)}} a \\ \beta^{un} &= \frac{\kappa_1 (1+2\tau) \sigma_u}{\sigma_v \sqrt{M\kappa_1 (1+2\tau)^2 [\kappa_1 (1-\tau) + 2\kappa_2 \tau] + N (\kappa_1 + 2\kappa_2 \tau)^2 (1+\tau)}}; \end{aligned}$$

for the realistic traders

$$\beta^r = \frac{(\kappa_1 + 2\kappa_2 \tau) \sigma_u}{\sigma_v \sqrt{M\kappa_1 (1+2\tau)^2 [\kappa_1 (1-\tau) + 2\kappa_2 \tau] + N (\kappa_1 + 2\kappa_2 \tau)^2 (1+\tau)}};$$

for the market maker

$$\begin{aligned} \mu &= -\frac{2(1+2\tau)\kappa_2 \tau}{(2\tau + N + 1)(\kappa_1 + 2\kappa_2 \tau) + M\kappa_1 (2\tau + 1)} a, \\ \lambda &= \frac{\sigma_v \sqrt{M\kappa_1 (1+2\tau)^2 [\kappa_1 (1-\tau) + 2\kappa_2 \tau] + N (\kappa_1 + 2\kappa_2 \tau)^2 (1+\tau)}}{\sigma_u [(2\tau + N + 1)(\kappa_1 + 2\kappa_2 \tau) + M\kappa_1 (2\tau + 1)]}. \end{aligned}$$

Proof. See Appendix. ■

When $\kappa_1 = \kappa_2$, the equilibrium described above is identical to the one found in proposition 1. As long as the unrealistic traders misperceive both variances by the same amount, the effects of the misperceptions onto $E_{un} [\tilde{v} | s = s_j]$ cancel each other. In all other cases when $\kappa_1 \neq \kappa_2$, both misperceptions affect the variables of the model.

We now interpret and discuss the equilibrium condition (1). The existence of the equilibrium, for $\kappa_1 \neq \kappa_2$, is subject to condition (1). Whenever that

[15] We have also solved the model where the two types of unrealistic traders, namely optimistic and pessimistic traders, trade together with the realistic traders and the liquidity traders. The results are qualitatively similar to the results obtained in that section. The proofs are available upon request from the authors.

condition is not verified a market breakdown occurs. If unrealistic traders distort their information revelation by increasing their trading intensity on private information, the equilibrium might fail to exist and a market breakdown might occur. Any misperception leading to a decrease of their trading intensity will not imply a market breakdown. If $\tau < 1$, unrealistic traders have little scope to increase their trading intensity as their private information is already very precise and an equilibrium exists. If $\tau > 1$, unrealistic traders have more scope to increase their trading intensity, however this negative effect is limited when the number of unrealistic traders is low relative to the number of realistic traders and/or if κ_1 is low relative to κ_2. Indeed, κ_1 and κ_2 have countervailing effects on β^{un} and therefore when the unrealistic traders are optimistic being sufficiently "pure optimist" alleviates the impact of being very overconfident. The same thing happens for the case of pessimistic unrealistic traders. In that case being too "pure pessimist" (κ_1 high) might lead to a market breakdown. Those effects are summarized in the following figure.

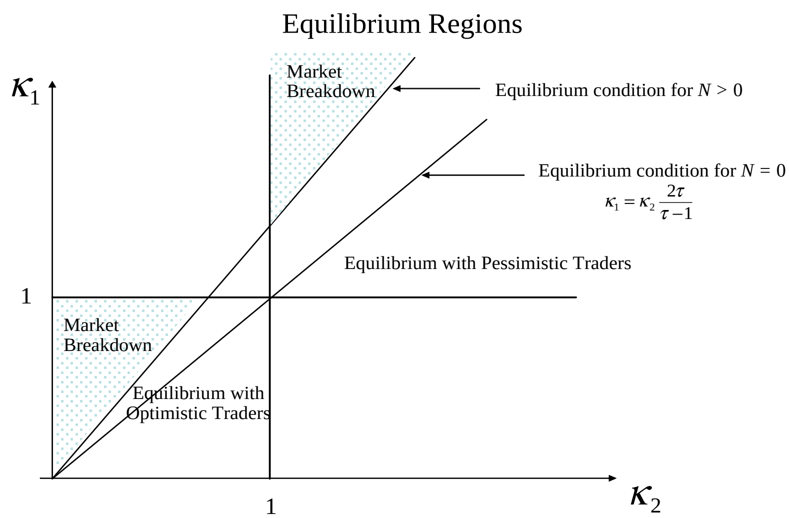

Figure 2: Equilibrium regions, as a function of κ_1 and κ_2, for $N = 0$ whenever $\tau > 1$ and for $N > 0$ whenever $\tau > 1$ and $\frac{M}{N} > \frac{1+\tau}{(\tau-1)(1+2\tau)^2}$.

In the preceding graph, combinations of κ_1 and κ_2 lying above the binding equilibrium condition lead to a market breakdown. This is shown for $N > 0$ and for $N = 0$.[16] The condition for $N = 0$ is more restrictive than the one for $N > 0$.

We now look at the traders' trading behavior derived in the above proposition. This is done in the two following lemmas. In the first one, we derive some comparative statics whereas in the second one, we compare the equilibrium obtained in proposition 3 to the one obtained in proposition 1.

[16] If $N = 0$ and provided M and κ_1 are positive, the equilibrium condition can be simplified to $\kappa_1 (1 - \tau) + 2\tau\kappa_2 \geq 0$.

Lemma 1 *Comparative Statics on Traders' Behavior*
Provided the equilibrium in Proposition 3 *exists, we have*

1. β^{un} *increases with* κ_1 *and decreases with* κ_2*,*
2. α^{un} *may increase or decrease with* κ_1 *and* κ_2*,*
3. *when* $(1-\tau)\,\kappa_1 + 2\tau\kappa_2 > 0$ β^r *decreases with* κ_1 *whereas it increases with* κ_1 *if* $(1-\tau)\,\kappa_1 + 2\tau\kappa_2 < 0$*; moreover when* $(1-2\tau)\,\kappa_1 + 2\tau\kappa_2 > 0$*,* β^r *increases with* κ_2 *and decreases with* κ_2 *if* $(1-2\tau)\,\kappa_1 + 2\tau\kappa_2 < 0$*.*

Proof. Straightforward by taking the expressions obtained in Proposition 3 and differentiating these expressions by the relevant parameters. ■

We now interpret and discuss Lemma 1. The unrealistic trading intensity on private information increases with κ_1 and decreases with κ_2. As prior information is misbelieved to be noisier and/or the noise in the signal is perceived to be smaller, the unrealistic trades more on private information. The response by the realistic traders to the unrealistic traders' behavior depends on the relative precision of prior information to the noise in the signal and on the relative level of error made on the two variances. If prior information is relatively less precise than the noise in private information ($\tau = \frac{\sigma_\varepsilon^2}{\sigma_v^2} \leq 1$) or if the error made on σ_v^2 is relatively smaller than the one on σ_ε^2 ($(1-\tau)\,\kappa_1 + 2\tau\kappa_2 > 0$), the realistic response is as expected, i.e., he decreases his information revelation with κ_1 and increases it with κ_2 in order to reduce the impact of his market order onto the price. As prior information becomes more precise relative to the noise in the private signal ($\tau > 1$), and κ_1 is large relative to κ_2, the realistic trader's information revelation increases with κ_1 and decreases with κ_2, following the unrealistic's behavior. This is, indeed, the case when $\kappa_1 > \kappa_2 \frac{2\tau}{2\tau-1}$. In that case, the unrealistic trader is trading more intensely on his private information than the realistic trader implying a large impact on price. As a result the realistic trader's marginal impact is smaller allowing him to behave as the unrealistic trader.

The following Lemma compares the level of trading intensities obtained in proposition 3, β^{un} and β^r, with the one obtained in proposition 1, β^{AP}, as well as the level of liquidity for proposition 3, λ^{un}, with the one of proposition 1, λ^{AP}.[17]

Lemma 2 *Provided the equilibrium in Proposition* 3 *exists, we have*

1. *when* $\kappa_1 < \kappa_2$*,* $\beta^r > \beta^{AP} > \beta^{un}$*, and* $\lambda^{un} > \lambda^{AP}$*,*
2. *when* $2\tau\,(1+\tau)\,\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 > 0$ *and* $\kappa_1 > \kappa_2$*,* $\beta^{un} > \beta^{AP} > \beta^r$*, and* $\lambda^{AP} > \lambda^{un}$*,*

[17] The superscript AP stands for the fact that the trading intensity and the liquidity parameter, in proposition 1, are equal to the ones found in Admati and Pfleiderer (1988).

3. *when* $2\tau(1+\tau)\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 < 0$, $\beta^{un} > \beta^r > \beta^{AP}$, *and* $\lambda^{AP} \lesseqqgtr \lambda^{un}$.

Proof. Straightforward. ■

We now discuss the above Lemma. When $\kappa_1 > \kappa_2$ ($\kappa_1 < \kappa_2$), the unrealistic trader's responsiveness to private information, s_j, is always higher (lower) than the realistic one. If the unrealistic trader is relatively more overconfident than optimistic or relatively more pessimistic than underconfident, he trades more intensely on his private information than his realistic counterpart. This may lead to a higher or lower liquidity than if all traders were realistic as in Admati and Pfleiderer (1988). A higher trading intensity on private information, on the one hand, decreases liquidity as it increases information revelation and, on the other hand increases it as it increases the variance of the aggregate order flow. A lower trading intensity leads to the two opposite effects on the liquidity. When the unrealistic trader is relatively more optimistic than overconfident or relatively more underconfident than pessimistic the liquidity is always lower than the one obtained in Admati and Pfleiderer.
The first two cases 1. and 2. of the above Lemma follow intuition. Indeed, traders are strategic and take into account the impact of their orders onto the price. Therefore, if the unrealistic traders' trading intensity is low (less than the one of proposition 1, β^{AP}), the realistic traders have some scope to increase their trading intensity. The overall impact of the trading intensities leads to a lower liquidity. If the opposite is true, the realistic traders scale down their trading intensity implying a more liquid market. The third case corresponds to the situation where both types of traders trade more intensely than in proposition 1, $\beta^{un} > \beta^r > \beta^{AP}$. However, the unrealistic traders' intensity is still larger than the realistic traders' one. Depending on their relative size, the liquidity is increased or decreased.

We now compute the traders' expected profit.

Proposition 4 *Provided the equilibrium exists, the expected profits are given by*

for the unrealistic traders

$$E\left(\Pi^{un}\right) = \frac{\sigma_v^2 (1+2\tau)^2 \kappa_1 \left[\kappa_1 (1-\tau) + 2\kappa_2\tau\right]}{\lambda\left((2\tau+N+1)(\kappa_1+2\kappa_2\tau) + M\kappa_1(2\tau+1)\right)^2},$$

for the realistic traders

$$E\left(\Pi^{r}\right) = \frac{\sigma_v^2 (1+\tau)\left[\kappa_1 + 2\kappa_2\tau\right]^2}{\lambda\left((2\tau+N+1)(\kappa_1+2\kappa_2\tau) + M\kappa_1(2\tau+1)\right)^2}.$$

Whenever $\kappa_1 < \kappa_2$, *the realistic traders earn strictly larger expected profit than unrealistic traders.*

Provided the equilibrium exists and that $\kappa_2 < \kappa_1$, *the unrealistic traders earn*

- *negative expected profit* $(\kappa_1(1-\tau) + 2\kappa_2\tau < 0)$,

- *positive expected profit however lower than the realistic one* $\left(\kappa_2 \frac{2\tau(1+\tau)}{2\tau^2-1} < \kappa_1 < \kappa_2 \frac{2\tau}{\tau-1}\right)$,

- *expected profit larger than the realistic one* $\left(2\tau\left(1+\tau\right)\kappa_2 - \left(2\tau^2-1\right)\kappa_1 > 0\right)$.

Proof. See Appendix. ■

The expected profits displayed in the preceding proposition are computed under the true distributions of $\tilde{v}$ and $\tilde{\varepsilon}$.

Given the equilibrium condition (1), four different situations arise depending on the value of both τ and the ratio $\frac{M}{N}$ for each case (optimistic or pessimistic): situation 1 is for $\tau \leq \frac{1}{\sqrt{2}}$, situation 2 for $\frac{1}{\sqrt{2}} < \tau \leq 1$, situation 3 for $1 < \tau$ and $\frac{M}{N} \leq \frac{1+\tau}{(1-\tau)(1+2\tau)^2}$, and situation 4 for $1 < \tau$ and $\frac{1+\tau}{(1-\tau)(1+2\tau)^2} < \frac{M}{N}$. We focus on situation 4 as it is the most complete. The graphs for the three other situations are put at the end of the paper (See figures 5, 6, and 7).

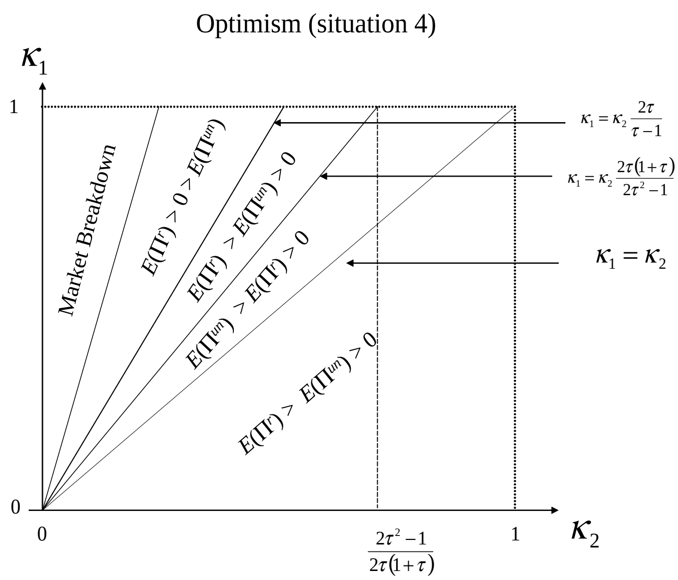

Figure 3: Expected Profit comparison for both a large τ $(1 < \tau)$ and a relatively high number of unrealistic traders in the market $(\frac{1+\tau}{(1-\tau)(1+2\tau)^2} < \frac{M}{N})$ with optimistic traders.

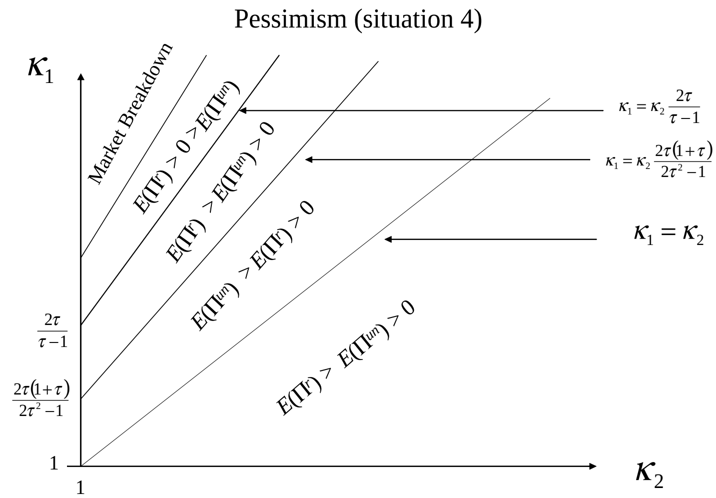

Figure 4: Expected Profit comparison for both a large τ $(1 < \tau)$ and a relatively high number of unrealistic traders in the market $(\frac{1+\tau}{(1-\tau)(1+2\tau)^2} < \frac{M}{N})$ with pessimistic.

We now comment on the above graphs. Whenever $\kappa_2 > \kappa_1$, the unrealistic traders always earn less expected profit than the realistic ones, however they are non-negative. In that case, unrealistic traders trade less aggressively on their private information than their rational counterpart and the market is less liquid than if all traders were realistic (liquidity in proposition 1).

For $\kappa_1 > \kappa_2$, depending on the relative value of κ_1 with respect to κ_2 and on the value of the other parameters (i.e. depending in which situation of the different ones cited above we are), the realistic traders may earn more expected profit than the unrealistic with the possibility for the latter to earn negative expected profit (situations 3 and 4), or the unrealistic traders may earn on average larger profits than the realistic.

When $2\tau(1+\tau)\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 < 0$, as seen above in the graph, both types of traders trade more aggressively on their private information, s_k, than they would if they were all realistic. The over-trading of the unrealistic is exacerbated by the realistic traders' over-trading. In that case and if both types of traders do not over-trade excessively, the unrealistic traders earn lower expected profit than the realistic traders, however non-negative. When both types do over-trade excessively, the unrealistic traders earn negative expected profit $(\kappa_1(1-\tau) + 2\kappa_2\tau < 0)$.

When the effect of unrealistic traders' over-trading is alleviated by the reduction in trading of the realistic traders, the unrealistic traders earn on average more than the realistic traders. This corresponds to the case where $2\tau(1+\tau)\kappa_2 - \left(2\tau^2 - 1\right)\kappa_1 > 0$ and $\kappa_1 > \kappa_2$.

For both cases (optimistic and pessimistic traders), as τ increases, the slope of the two lines ($\kappa_2 \frac{2\tau}{\tau-1}$ and $\kappa_2 \frac{2\tau(1+\tau)}{2\tau^2-1}$) becomes flatter implying that the region where the unrealistic trader earns more than the realistic trader shrinks. Ultimately, for an infinite τ and for the parameters where the equilibrium exists, when $\kappa_2 < \kappa_1$, the unrealistic trader earns negative expected profit and the realistic positive, whereas when $\kappa_1 < \kappa_2$ the unrealistic trader earns positive expected profit although lower than the realistic one. When τ is infinite, the unrealistic trader never earns profit, in expected terms, higher than the realistic trader.

The next Lemma shows the expression of the price efficiency as well as the one of the ex-ante volatility. It also shows their comparative statics with respect to κ_1 and κ_2.

Lemma 3 (Price Efficiency and Ex-ante Volatility) *Provided the equilibrium in Proposition* 3 *exists, we have*

- *Price efficiency is equal to*

$$var\left(v|\,p\right) = \frac{\sigma_v^2\left(\kappa_1 + 2\tau\kappa_2\right)\left(1+2\tau\right)}{\left(N+2\tau+1\right)\left(\kappa_1+2\tau\kappa_2\right)+M\kappa_1\left(2\tau+1\right)}.$$

  It increases with κ_2 *whereas it decreases with* κ_1*.*

- *The ex-ante volatility is equal to*

$$var\left(p\right) = \frac{\sigma_v^2\left(N\left(\kappa_1 + 2\tau\kappa_2\right)+M\kappa_1\left(2\tau+1\right)\right)}{\left(N+2\tau+1\right)\left(\kappa_1+2\tau\kappa_2\right)+M\kappa_1\left(2\tau+1\right)}.$$

  It increases with κ_1 *and decreases with* κ_2*.*

Proof. Straightforward. ■

We now interpret Lemma 3. The effect of κ_1 onto both the volatility and the price efficiency accords to intuition. However the effect of κ_2 deserves more attention. As κ_2 increases, the unrealistic traders trade less aggressively on their private information, s_j. As explained before, the realistic traders' reaction is not as clear. However, the overall effect is as stated in the Lemma.

Kyle and Wang (1997) explore the strategic interaction between investors. They focus on the case where one unrealistic, either overconfident or underconfident, investor competes against a realistic one. The unrealistic trader misperceives the variance of the noise of his/her own signal and has some beliefs concerning the variance of the noise of the other trader's signal. They obtain that the equilibrium only depends on the trader's misperception of the variance of the noise of his/her own signal. They find that when an unrealistic trader is able to credibly commit to trade a larger quantity, this trader can outperform the realistic trader. They show that an overconfident trader is able to commit whereas an underconfident is not. We do obtain the same result for an optimistic trader. However, some of our results concerning pessimistic traders are qualitatively different. In our model, a pessimistic trader believes that prior information has a greater variance. This enables him/her to commit to trade larger quantity. As a consequence, we find that pessimistic trader can outperform realistic traders.

4 Unrealistic Market Makers

We now look at the case where the market maker is unrealistic as well as M traders among the $M+N$ traders.[18]

The unrealistic traders misperceive the distributions of both $\tilde{v}$ and $\tilde{\varepsilon}_j$ as before. Given the fact that the market maker has no access to any private signal, she misperceives the expectation and variance of the distribution of prior information. The market maker believes that the distribution of the asset is such that

$$\tilde{v} \rightarrow N\left(\overline{a}, \overline{\kappa}_1 \sigma_v^2\right).$$

As before the market maker behaves competitively.

Proposition 5 *Whenever*

$$M\kappa_1 (1+2\tau)^2 \left[\kappa_1 (\overline{\kappa}_1 - \tau) + 2\kappa_2 \tau\right] + N (\kappa_1 + 2\kappa_2 \tau)^2 \left((2\tau+1)\overline{\kappa}_1 - \tau\right) \geq 0. \quad (2)$$

there exists a unique linear equilibrium. It is characterized by the following parameters,

for the optimistic/pessimistic traders

$$\begin{aligned}
\alpha^{un} &= \frac{(2\tau+1)(2\kappa_2\tau a - \overline{a}(\kappa_1+2\kappa_2\tau))\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\overline{\kappa}_1-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\overline{\kappa}_1-\tau)}}, \\
\beta^{un} &= \frac{\kappa_1(2\tau+1)\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\overline{\kappa}_1-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\overline{\kappa}_1-\tau)}},
\end{aligned}$$

for the realistic traders

$$\begin{aligned}
\alpha^{r} &= -\frac{\overline{a}(\kappa_1+2\kappa_2\tau)(2\tau+1)\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\overline{\kappa}_1-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\overline{\kappa}_1-\tau)}}, \\
\beta^{r} &= \frac{(\kappa_1+2\kappa_2\tau)\sigma_u}{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\overline{\kappa}_1-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\overline{\kappa}_1-\tau)}},
\end{aligned}$$

for the market maker

$$\begin{aligned}
\mu &= \frac{(2\tau+1)[\overline{a}(\kappa_1+2\kappa_2\tau)(M+N+1)-2M\kappa_2\tau a]}{M\kappa_1(2\tau+1)+N(\kappa_1+2\kappa_2\tau)+(2\tau+1)(\kappa_1+2\kappa_2\tau)}, \\
\lambda &= \frac{\sigma_v\sqrt{M\kappa_1(1+2\tau)^2[\kappa_1(\overline{\kappa}_1-\tau)+2\kappa_2\tau]+N(\kappa_1+2\kappa_2\tau)^2((2\tau+1)\overline{\kappa}_1-\tau)}}{M\kappa_1(2\tau+1)+N(\kappa_1+2\kappa_2\tau)+(2\tau+1)(\kappa_1+2\kappa_2\tau)\sigma_u}.
\end{aligned}$$

[18] The market maker, given her privileged position in the market, is usually thought to be realistic. However, allowing her to be unrealistic enables us to have the same effects as the ones we would obtain in a Grossman and Stiglitz type of model with an unrealistic liquidity supplier.

Proof. See Appendix. ■

We first comment on condition (2). From its expression, we see that an optimistic market maker exacerbates the occurrence of a market breakdown whereas a pessimistic one alleviates it. This can be explained as follows. In the following discussion, we only look at the effect of an optimistic market maker as the pessimistic case is symmetric. The market maker's optimism affects the price function in two opposite ways. On the one hand, the higher the misperception is the higher the market depth. Indeed, an optimistic market maker thinks that the prior information is more precise than it is and therefore believes that the informed private information is less substantial than it actually is. As a consequence, she adjusts her price less aggressively. This is done by reducing the liquidity parameter, λ, and therefore by increasing market depth. As a response, informed traders trade more intensely. As they trade more intensely, a market breakdown is more likely to occur. On the other hand, the overall level of price is shifted up due to the misperception of the expectation. Indeed, an optimistic market maker wrongly believes that the expectation of the risky asset is higher than it actually is and therefore increases the overall level of prices. As a response, traders reduce the size of their market order. That reduction is proportional to the misperception $\overline{a}$. However, the effect of $\overline{a}$ on the level of prices is mitigated by the effect of the trader's misperception of the expectation as seen in the equation defining μ. Whenever the market maker and the unrealistic traders hold opposite beliefs about the distributions, the shift is positive (negative) with an optimistic (pessimistic) market maker. When the market and the traders are all optimistic or pessimistic, the level of prices can either be increased ($\mu > 0$) or reduced ($\mu < 0$). The former happens when the market maker's optimism (pessimism) is relatively larger (smaller) then the traders' optimism (pessimism). The latter happens for all the converse cases.

In that case, the price sets by the market maker incorporates her unrealism possibly leading to an irrational bubble. Indeed, when the market maker is unrealistic and due to her misperception of the risky asset's misperception, she increases the overall level of prices. This can be understood as the early stage of a price bubble. However, our model being static, we cannot perform a dynamic analysis of that bubble.

The unconditional expected profits of each traders and for the market maker are given in the following proposition.

Proposition 6 *Provided the equilibrium exists, the expected profits are given by*

for the unrealistic traders

$$E\left[\Pi^{un}\right] = \tfrac{(2\tau+1)^2}{\lambda d^2}\left[\sigma_v^2\kappa_1\left(\kappa_1\left(1-\tau\right)+2\kappa_2\tau\right)-\overline{a}\left(\kappa_1+2\kappa_2\tau\right)\left(2\kappa_2\tau a-\left(\kappa_1+2\kappa_2\tau\right)\overline{a}\right)\right],$$

for the realistic traders

$$E\left[\Pi^{r}\right] = \tfrac{(\kappa_1+2\kappa_2\tau)^2}{\lambda d^2}\left[\sigma_v^2\left(\tau+1\right)+\overline{a}^2\left(2\tau+1\right)^2\right],$$

for the market maker

$$\begin{aligned} E\left[\Pi^{MM}\right] &= \tfrac{(2\tau+1)(\kappa_1+2\kappa_2\tau)}{\lambda d^2}\left[\sigma_v^2\left(\bar{\kappa}_1-1\right)\left(M\kappa_1\left(2\tau+1\right)+N\left(\kappa_1+2\kappa_2\tau\right)\right)\right. \\ &\left. +\overline{a}\left(2\tau+1\right)\left(2M\kappa_2\tau a-\overline{a}\left(\kappa_1+2\kappa_2\tau\right)\left(M+N\right)\right)\right]. \end{aligned}$$

Proof. See Appendix. ■

The expected profit for both types of traders is affected by the market marker's misperception on both the expected return of the asset and the variance. The lower the perceived variance (the lower $\bar{\kappa}_1$), the higher the market depth, and therefore the higher the trader's unconditional expected profit. The misperception of the expectation affects differently the two types of traders. The realistic trader's unconditional expected profit increases with it. The analysis for the unrealistic's unconditional expected profit is not as straightforward. Whenever α^{un} and $\bar{a}$ have different sign, the unrealistic trader's expected profit is larger. This happens when the market maker and the unrealistic traders hold opposite beliefs regarding the distributions or may happen when the market maker and the unrealistic traders are all optimistic or all pessimistic. As before, the unrealistic trader may obtain greater, equal or smaller profit than the realistic one with the possibility for him to have negative expected profit.

The market maker, when realistic ($\bar{\kappa}_1 = 1$, $\overline{a} = 0$), obtains zero expected profit. However, if she is unrealistic, either optimistic or pessimistic, she may obtain expected profit different from zero. An increase of $\overline{\kappa}_1$ has several effects on the market maker's expected profit. An increase of $\overline{\kappa}_1$ increases λ which in turn decreases the aggregate order flow. The magnitude of these countervailing effects determine the behavior of the market maker's expected profit. The effect of the additive misperception depends on the sign of the traders' additive misperception. If the market maker and the traders hold opposite beliefs the expected profit is decreased whereas if they are all pessimistic or optimistic, the market maker's expected is either increased or decreased.

Lemma 4 (Price Efficiency and Ex-ante Volatility) *Provided the equilibrium in Proposition* 5 *exists, we have*

- *The ex-ante volatility is equal to*

$$var\left(p\right)=\tfrac{\sigma_v^2(N(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1))((\kappa_1+2\tau\kappa_2)(N+\bar{\kappa}_1(2\tau+1))+M\kappa_1(2\tau+1))}{((N+2\tau+1)(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1))^2}.$$

  It increases with $\bar{\kappa}_1$.

- *The price efficiency is given by*

$$var\left(v|\,p\right)=\tfrac{\sigma_v^2\bar{\kappa}_1(\kappa_1+2\tau\kappa_2)(1+2\tau)}{(N+\bar{\kappa}_1(2\tau+1))(\kappa_1+2\tau\kappa_2)+M\kappa_1(2\tau+1)}.$$

  It increases with $\bar{\kappa}_1$.

Proof. Straightforward. ■

Both the ex-ante volatility and the price efficiency increase with $\overline{\kappa}_1$. This implies that both are lower with an optimistic market maker than with a pessimistic one. As described before, an optimistic (pessimistic) market maker sets prices less (more) aggressively by increasing (decreasing) liquidity, traders respond to it by increasing (decreasing) their trading intensity. For the ex-ante volatility, the effect on the market depth dominates the effects on the trading intensity. Regarding now the price efficiency, on the one hand an increase of $\overline{\kappa}_1$ decreases the traders' trading intensity and on the other hand it increases volatility. As the volatility effect always dominates, the price efficiency increases with $\overline{\kappa}_1$.

5 Conclusion

We develop, here, a model of optimism and pessimism in financial markets. We model unrealistic traders (optimistic/pessimistic) as traders who, as well as misperceiving the expected returns of the asset, misperceive the variance of both the volatility of the asset returns and the noise in the private signal. An optimistic (pessimistic) trader over-estimates (under-estimates) the expected returns of the asset and under-estimates (over-estimates) both variances. We study two scenarios, in the first one the unrealistic trader only misperceives the expected returns whereas in the second one, the unrealistic trader misperceives the expected returns and the variance of both the volatility of the asset returns and the noise in the private signal. Moreover, in the second scenario, the marker maker is realistic (in the first part) and then unrealistic. In scenario 1, we find that an optimistic (pessimistic) trader purchases (sells) a larger quantity or sells (purchases) a smaller one. We show that the liquidity is not affected by the misperception of the expected returns and is equal to the one we would obtain if all traders were realistic. This is due to the fact that unrealistic traders alter the size of their market order through the misperception of the returns of the asset, without affecting their information revelation. As a consequence, the aggregate order flow faced by the market maker conveys the same amount of information as if all traders were realistic. The expected profit for the unrealistic trader and for the realistic trader are shown to be equal. It is not costly to be unrealistic. In scenario 2, we show that a market breakdown can occur. This is indeed the case when both types of traders trade excessively on their private information. If the impact of the unrealistic traders' over-trading is reduced by the realistic traders' trading behavior, the unrealistic traders' expected profit is larger than the realistic one. However, if the impact of that over-trading is not sufficiently reduced by the realistic traders or if the realistic traders trade more intensely on private information than the unrealistic traders, the realistic traders earn on average larger profit than the unrealistic traders. Finally we show that the price efficiency improves with κ_2 whereas it decreases with κ_1, and that the volatility decreases with κ_2 and increases with κ_1.

When the market maker is unrealistic, we, first of all, show that an optimistic market maker exacerbates the occurrence of a market breakdown whereas a pessimistic market maker alleviates it. The introduction of an unrealistic market maker may affect differently the traders. Realistic traders have larger expected profits when the market maker is unrealistic. We, finally, show that when the market maker is unrealistic, her expected profit may be either positive or negative.

An interesting extension of the model would be to look at how the results obtained in the present model would be modified in a dynamic setting. A further extension of the present model could include different forms of unrealism. This is left for future research.

6 Bibliography

Admati, A., and P. Pfleiderer, 1988, “A Theory of Intraday Patterns: Volume and Price Variability,” *Review of Financial Studies*, Vol. 1, No. 1, pp 3-40.

Barber, B. M., and T. Odean, 2000, “Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors,” *Journal of Finance*, Vol. 55, No. 2, pp 773-806.

Barber, B. M., and T. Odean, 2001, “Boys will be Boys: Gender, Overconfidence and Common Stocks Investments,” *Quarterly Journal of Economics*, Vol. 116, pp 261-292.

Barber, B. M., and T. Odean, 2002, “Online Investors: Do The Slow Die First?,” *Review of Financial Studies*, Vol. 15, No. 2, pp 445-487.

Biais, B., D. Hilton, K. Mazurier, and S. Pouget, 2005, “Judgemental Overconfidence, Self-Monitoring and Trading Performance in an Experimental Financial Market,” *Review of Economic Studies*, Vol. 72, No. 2.

Bénabou, R., and J. Tirole, 2002, “Self-confidence and Personal Motivation,” *The Quarterly Journal of Economics*, Vol. 117 pp 871-913.

Benos, A., 1998, “Aggressiveness and Survival of Overconfident Traders,” *Journal of Financial Markets*, Vol. 1, pp 353-383.

Bloomfield, R., R. Libby and M. W. Nelson, 2000, “Underreactions, Overreactions and Moderated Confidence,” *Journal of Financial Markets*, Vol. 3, pp 113-137.

Brocas, I., and J. Carrillo, 2004, “Entrepreneurial Boldness and Excessive Investment,” *Journal of Economics and Management Strategy*, Vol. 21, pp 321-350.

Caballé, J., and J. Sákovics, 2003, “Speculating Against an Overconfident Market,” *Journal of Financial Markets*, Vol. 6, pp 199-225.

Cabantous, L., D. Hilton, I. Régner, and S. Vautier, 2004, “Judgmental Overconfidence: One Positive Illusion or Many?,” Mimeo Toulouse.

Chauveau, T., and N. Nalpas, 2004, “A Theory of Disappointment Weighted Utility,” Mimeo Toulouse Business School.

Chauveau, T., and N. Nalpas, 2004, “Disappointment, Pessimism and the Equity Risk Premia,” Mimeo Toulouse Business School.

Cornelli., F., D. Goldreich and A. Ljungqvist, 2005, ”Investor Sentiment and Pre-IPO markets,” *Journal of Finance*, Forthcoming.

Daniel, K., D. Hirshleifer and A. Subrahmanyam, 1998, “Investor Psychology and Security Market Under-and Overreactions,” *Journal of Finance*, Vol. 53, No. 6, pp 1839-1885.

Daniel, K., D. Hirshleifer and A. Subrahmanyam, 2001, “Overconfidence, Arbitrage, and Equilibrium Asset Pricing,” *Journal of Finance*, Vol. 56, pp 921-965.

Daniel, K., D. Hirshleifer and S. H. Teoh, 2002, “Investor Psychology in Capital Markets: Evidence and Policy Implications,” *Journal of Monetary Economics*, Vol. 49, pp 139-209.

Debondt, W., and R. Thaler, 1985, “Does the Stock Market Overreact?” *Journal of Finance*, Vol. 40, pp 793-805.

De Mezza, D., and C. Southey, 1996, "The Borrower's Curse: Optimism, Finance and Entrepreneurship," *The Economic Journal*, Vol. 106, No. 435, pp 375-386.

Didri, R., and L. Germain, 2006, "Noise and Competition in Strategic Oligopoly," Working Paper Toulouse Business School.

Dow, J., and G. Gorton, 1997, "Noise Trading, Delegated Portfolio Management, and Economic Welfare," *Journal of Political Economy*, Vol. 105, No. 5, pp 1024-1050.

Forsythe, R., T. Rietz and T. Ross, 1999, "Wishes, Expectations and Actions: a Survey on Price Formation in Election Stock Markets," *Journal of Economic Behavior and Organization*, Vol. 39, pp 83-110.

Garber, P., 2000, *Famous First Bubbles The Fundamental Of Early Manias*, The MIT Press.

Gervais, S., and T. Odean, 2001, "Learning to be Confident," *Review of Financial Studies*, Vol. 14, No. 1, pp 1-27.

Harris, and Middleton, 1994, "The illusion of control and optimism about health: On being less at risk but no more in control than others," *British Journal of Social Psychology,* 33, 369-386.

Heifetz, A., and Y. Spiegel, 2001, "The Evolution of Biased Perceptions", Mimeo.

Hilton, D., 2001, "Psychology and the Financial Markets: Applications to Trading, Dealing and Investment Analysis," *Psychology and the Financial Markets*, pp 37-53.

Hoorens, V., 2001, "Why do Controllable Events Elicit Stronger Comparative Optimism than Controllable Events," *International Review of Social Psychology.*

Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica*, Vol. 53, No. 6, pp 1315-1336.

Kyle, A. S., and F. A. Wang, 1997, "Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test," *Journal of Finance*, Vol. 52, No. 5, pp 2073-2090.

Langer, E., (1975), "The Illusion of Control," *Journal of Personality and Social Psychology*, Vol. 32, pp. 311-328.

Manove, M., and J. Padilla, 1999, "Banking (Conservatively) with Optimists," *RAND Journal of Economics*, Vol. 30, No. 2, pp 324-350.

Manove, M., 2000, "Entrepreneurs, Optimism and the Competitive Edge," UFAE and IAE Working Papers 296.95.

Odean, T., 1998a, "Are Investors Reluctant to Realize their Losses?," *Journal of Finance*, Vol. 53, No. 5, pp 1775-1798.

Odean, T., 1998b, "Volume, Volatility, Price, and Profit When all Traders are Above Average," *Journal of Finance*, Vol. 53, No. 6, pp 1887-1924.

Odean, T., 1999, "Do Investors Trade Too Much," *American Economic Review*, Vol. 89, No. 5, pp 1279-1298.

Régner, I., D. Hilton, L. Cabantous, and S. Vautier, 2004, "Judgmental Overconfidence: One Positive Illusion or Many?," Mimeo.

Ritter, J., 1991, "The Long-Run Performance of Initial Public Offerings," *Journal of Finance*, Vol. 46, No. 1, pp 3-27.

Scheinkman, J. A., and W. Xiong, 2003, "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, Vol. 111, No. 6, pp 1183-1219.

Shiller, R., 1981, "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?," *American Economic Review*, Vol. 71, pp 421-435.

Shiller, R., 1989, "Comovements in Stock Prices and Comovements in Dividends," *Journal of Finance*, Vol. 44, No. 3, pp 719-729.

Shiller, R., 2000, *Irrational Exuberance*, Princeton University Press.

Sutherland, N. S., 1992, "*Irrationality: The Enemy Within*," Harmondsworth: Penguin.

Taylor, S., and J. Brown, 1988, "Illusion and Well-being: A Social Psychological Perspective on Mental Health," *Psychological Bulletin*, Vol. 103, pp 193-210.

Taylor, S., and J. Brown, 1994, "Positive Illusion and Well-being Revisited Separating Fact From Fiction," *Psychological Bulletin*, Vol. 116, pp 21-27.

Van Den Steen, E., 2004, "Rational Overoptimism (and Other Biases)," *American Economic Review*, Vol. 94, September, pp 1141-1151.

Weinstein, N., 1980, "Unrealistic Optimism About Future Life Events," *Journal of Personality and Social Psychology*, Vol. 39, pp 806-820.

Womack, K., 1996, "Do Brokerage Analysts' Recommendations Have Investment Values," *Journal of Finance*, Vol. 51, No. 1, pp 137-167.

7 Appendix

7.1 Proofs

Proof of Proposition 1 (Equilibrium for Additive Misperception) Take the results obtained in proposition 3 for the expression of the parameters α^{un}, β^{un}, α^{r}, β^{r}, μ, and λ and set $\kappa_1 = 1$ and $\kappa_2 = 1$.

Proof Proposition 2 (Expected Profits for Additive Misperception) Take the results obtained in proposition 4 for the expression of both profits and set $\kappa_1 = 1$ and $\kappa_2 = 1$.

Proof of Proposition 3 (Equilibrium for Additive and Multiplicative Misperception)

Given the expressions of the market orders submitted by the optimistic traders, x^o, and by the realistic traders, x^r, the aggregate order flow is equal to

$$y = \sum_{i=1}^{N} x_i^r + \sum_{j=1}^{M} x_j^{un} + u = \left(N\beta^r + M\beta^{un}\right) v + \beta^r \sum_{i=1}^{N} \varepsilon_i + \beta^{un} \sum_{j=1}^{M} \varepsilon_j + N\alpha^r + M\alpha^{un} + u.$$

The unrealistic trader maximizes his conditional expected profit

$$\max_{x_j^{un}} E_{un}\left(\left(v-p\right) x_j^{un} \middle| s = s_j \right).$$

Substituting the form of the price as well as the market orders form for the N realistic traders, and the $M-1$ optimistic in the above expression, computing the first order condition and solving it for the market order, we obtain

$$\begin{aligned} x_j^{un} &= \frac{1}{2\lambda}\left[E_{un}\left(v|\, s=s_j\right)\left(1-(M-1)\lambda\beta^{un}-N\lambda\beta^r\right)\right. \\ &\left. -\mu-(M-1)\lambda\alpha^o-N\lambda\alpha^r\right]. \end{aligned} \tag{3}$$

We now need to compute $E_{un}\left(v|\, s=s_j\right)$. On one side and given the normality of the random variables we have that $E_{un}\left(v|\, s=s_j\right)=\gamma\left(s_j-E_{un}(v)\right)+E_{un}(v)$ with $\gamma=\frac{cov_{un}(v,s_j)}{var_{un}[s_j]}$. Given that $E_{un}(v)=a$ and $\tau=\frac{\sigma_\varepsilon^2}{\sigma_v^2}$, we obtain

$$E_{un}\left(v|\, s=s_j\right)=\frac{\kappa_1}{\kappa_1+\kappa_2\tau}s_j+a\frac{\kappa_2\tau}{\kappa_1+\kappa_2\tau}.$$

Replacing the expression of the conditional expectation into the form of the order (3) and identifying the parameters we have

$$\beta^{un} = \frac{\kappa_1\left(1-\lambda N\beta^r\right)}{\lambda\left((M+1)\kappa_1+2\kappa_2\tau\right)}, \tag{4}$$

$$\begin{aligned} \alpha^{un} &= \frac{1}{\lambda(M+1)}\left[a\frac{\kappa_2\tau}{\kappa_1+\kappa_2\tau}\left(1-(M-1)\lambda\beta^{un}-N\lambda\beta^r\right)\right. \\ &\left. -\mu-\lambda N\alpha^r\right]. \end{aligned} \tag{5}$$

The second order condition is satisfied.

Finally, the realistic maximizes his conditional expected profit. Given his first order condition and the fact that $E\left(v|\, s=s_i\right)=\frac{s_i}{1+\tau}$, the parameters for the realistic's market order are such that

$$\beta^r = \frac{\left(1-\lambda M\beta^{un}\right)}{\lambda\left(N+1+2\tau\right)}, \tag{6}$$

$$\alpha^r = -\frac{1}{\lambda(N+1)}\left[\mu+\lambda M\alpha^{un}\right]. \tag{7}$$

The second order condition is satisfied.

The market maker behaves competitively and sets a price such that

$$p=E\left[v|\, y\right]=0+\frac{cov(v,y)}{var[y]}\left(y-E(y)\right).$$

Given the expression of the aggregate order flow, the parameters of the price schedule are given by

$$\lambda = \frac{\left(N\beta^r+M\beta^{un}\right)}{\left(N\beta^r+M\beta^{un}\right)^2+\left(N\left(\beta^r\right)^2+M\left(\beta^{un}\right)^2\right)\tau+\frac{\sigma_u^2}{\sigma_v^2}}, \tag{8}$$

$$\mu = -\lambda\left(N\alpha^r+M\alpha^{un}\right). \tag{9}$$

Solving the above system of six equations defined by equations (4), (5), (6), (7), (8), and (9) for the six unknowns leads to the result of proposition 4.

Proof of Proposition 4 (Expected Profits for Additive and Multiplicative Misperception)

The expected profit of any trader, $h = un$ or r, can be written as

$$E\left(\Pi^h\right) = E\left((v-p)\,x^h\right) = E\left((v-\mu-\lambda y)\left(\beta^h\left(v+\varepsilon_h\right)+\alpha^h\right)\right).$$

Given the expression of y and after simplification of some of the terms, the expected profit is equal to

$$E\left(\Pi^h\right) = E\left[\left(v-\lambda\left(\left(N\beta^r+M\beta^o\right)v+\beta^r\sum_{i=1}^{N}\varepsilon_i+\beta^{un}\sum_{j=1}^{M}\varepsilon_j+u\right)\right)\left(\beta^h\left(v+\varepsilon_h\right)+\alpha^h\right)\right].$$

All random variables are independent and have a zero mean, we therefore get

$$E\left(\Pi^h\right) = E\left(v^2\beta^h\left(1-\lambda\left(N\beta^r+M\beta^{un}\right)\right)-\lambda\beta^h\varepsilon^h\left(\beta^r\sum_{i=1}^{N}\varepsilon_i+\beta^{un}\sum_{j=1}^{M}\varepsilon_j\right)\right).$$

This expression simplifies to

$$\begin{aligned}
E\left(\Pi^{un}\right) &= \beta^{un}\left[\sigma_v^2\left(1-\lambda\left(N\beta^r+M\beta^{un}\right)\right)-\lambda\beta^{un}\sigma_\varepsilon^2\right] \text{ for the unrealistic trader,}\\
E\left(\Pi^h\right) &= \beta^r\left[\sigma_v^2\left(1-\lambda\left(N\beta^r+M\beta^{un}\right)\right)-\lambda\beta^r\sigma_\varepsilon^2\right] \text{ for the realistic trader.}
\end{aligned}$$

Using the expressions of β^r, and β^{un} and after some simplifications we obtain for each type of traders

$$\begin{aligned}
E\left(\Pi^{un}\right) &= \frac{\sigma_v^2\left(1+2\tau\right)^2\kappa_1\left[\kappa_1\left(1-\tau\right)+2\kappa_2\tau\right]}{\lambda\left(\left(2\tau+N+1\right)\left(\kappa_1+2\kappa_2\tau\right)+M\kappa_1\left(2\tau+1\right)\right)^2},\\
E\left(\Pi^r\right) &= \frac{\sigma_v^2\left(1+\tau\right)\left[\kappa_1+2\kappa_2\tau\right]^2}{\lambda\left(\left(2\tau+N+1\right)\left(\kappa_1+2\kappa_2\tau\right)+M\kappa_1\left(2\tau+1\right)\right)^2}.
\end{aligned}$$

Having the expression of the expected profit for unrealistic traders and for realistic traders, we can compare them to each other. We compute the difference in expected profits, after some rearrangements that leads to

$$E\left(\Pi^{un}\right)-E\left(\Pi^r\right) = \frac{\kappa_1\left(1+2\tau\right)\sigma_v^2}{\lambda d^2}\left(\left(1+2\tau\right)^2\kappa_1\left[\kappa_1\left(1-\tau\right)+2\kappa_2\tau\right]-\left(1+\tau\right)\left[\kappa_1+2\kappa_2\tau\right]^2\right),$$

where $d = \left(2\tau+N+1\right)\left(\kappa_1+2\kappa_2\tau\right)+M\kappa_1\left(2\tau+1\right)$.

Given the above expression, finding the sign of $E\left(\Pi^{un}\right)-E\left(\Pi^r\right)$ is equivalent to find the sign of

$$\left(1+2\tau\right)^2\kappa_1\left[\kappa_1\left(1-\tau\right)+2\kappa_2\tau\right]-\left(1+\tau\right)\left[\kappa_1+2\kappa_2\tau\right]^2.$$

It is straightforward to prove that the previous expression is equal to

$$2\tau\left(\kappa_1-\kappa_2\right)\left[\kappa_1\left(1-2\tau^2\right)+2\kappa_2\left(1+\tau\right)\tau\right]. \tag{10}$$

Whenever $\tau \leq \frac{1}{\sqrt{2}}$, the expression (10) is of the sign of $\kappa_1-\kappa_2$ and when $\kappa_1-\kappa_2>0$ (<0), we have $E\left(\Pi^r\right)<E\left(\Pi^{un}\right)$ $\left(E\left(\Pi^{un}\right)<E\left(\Pi^r\right)\right)$. Whenever $\frac{1}{\sqrt{2}}<\tau$, (10) has two positive roots $\kappa_1=\kappa_2$ and $\kappa_1=\frac{2\kappa_2(1+\tau)\tau}{2\tau^2-1}\kappa_2$. One can prove that the latter is always greater than the former. For any κ_2 and for κ_1 in the interval $\left[\kappa_2,\frac{2\kappa_2(1+\tau)\tau}{2\tau^2-1}\kappa_2\right]$ we have $E\left(\Pi^r\right)<E\left(\Pi^{un}\right)$, for any κ_2 and for κ_1 outside the interval we obtain that $E\left(\Pi^{un}\right)<E\left(\Pi^r\right)$. Given the expression of the unrealistic's expected profit, one can see that if $\tau>1$ and $\kappa_1>\frac{2\tau}{\tau-1}\kappa_2$, unrealistic traders earn negative expected profits.

Proof of Proposition 5 (Unrealistic Market Makers)

After maximizing the traders expected utility we get for the different parameters

$$\begin{aligned}
\alpha^{un} &= -\frac{1}{\lambda\left(M+1\right)}\left[\frac{\kappa_{2\tau}}{\kappa_1+\kappa_2\tau}\left(1-\lambda\left(M-1\right)\beta^{un}-\lambda N\beta^r\right)\right], && (11)\\
\beta^{un} &= \frac{\kappa_1\left(1-\lambda N\beta^r\right)}{\lambda\left(\left(M+1\right)\kappa_1+2\tau\kappa_2\right)} \\
\alpha^r &= -\frac{1}{\lambda\left(N+1\right)}\left[\mu+\lambda M\alpha^{un}\right] && (12)\\
\beta^r &= \frac{1-\lambda M\beta^{un}}{\lambda\left(N+1+2\tau\right)}.
\end{aligned}$$

The market maker sets a price, p, such that

$$p=\bar{E}\left[\tilde{v}\right|y]=\bar{E}\left[\tilde{v}\right]+\frac{\overline{cov}\left(\tilde{v},y\right)}{\overline{var}\left(y\right)}\left(y-\bar{E}\left(y\right)\right),$$

where the upper bar denotes that the expectation, covariance and variance are computed given the wrong beliefs of the market maker.

Given the market maker's additive misperception we obtain

$$\begin{aligned}
\lambda &= \frac{\left(M\beta^{un}+N\beta^r\right)\overline{\kappa}_1}{\left(M\beta^{un}+N\beta^r\right)^2\overline{\kappa}_1+\left(M\beta^{un^2}+N\beta^{r^2}\right)\tau+\frac{\sigma_u^2}{\sigma_v^2}}, && (13)\\
\mu &= \left(1-\lambda M\beta^{un}-\lambda N\beta^r\right)\overline{a}-\lambda M\alpha^{un}-\lambda N\alpha^r. && (14)
\end{aligned}$$

Solving the above system of six equations with six unknowns leads to the desired result.

Proof of Proposition 6 (Expected Profit) Follow the same steps as in proposition 4 for the expected profits of the traders.

The market maker's expected profit are equal to

$$E\left[\Pi^{MM}\right]=-NE\left(\Pi^r\right)-ME\left(\Pi^{un}\right)+E\left(\Pi^{Liq}\right).$$

It is straightforward to show that the expected profit of the liquidity traders, $E\left(\Pi^{Liq}\right)$, are equal to $\lambda\sigma_u^2$. Plug the expressions found for the two types of traders and for the liquidity traders into the expression above and after some manipulations, the desired result is found.

7.2 Figures: Expected Profit Comparison

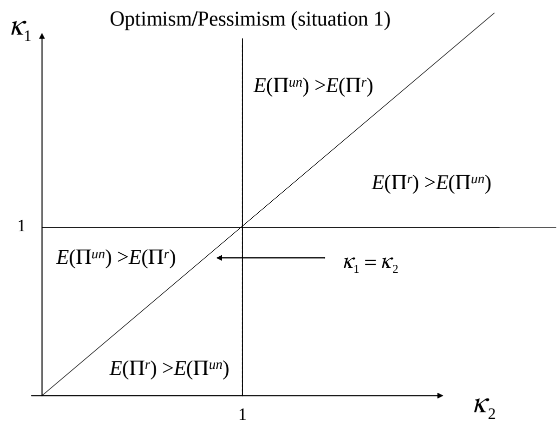

Figure 5: Expected Profit comparison for a low τ ($\tau \leq \frac{1}{\sqrt{2}}$) with optimists/pessimists.

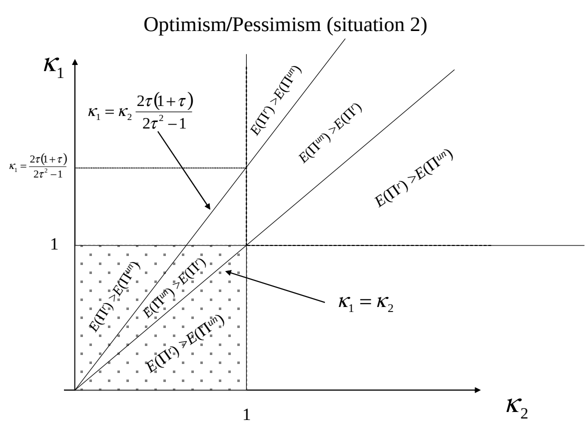

Figure 6: Expected Profit comparison for an intermediate τ ($\frac{1}{\sqrt{2}} < \tau \leq 1$) with optimists/pessimists.

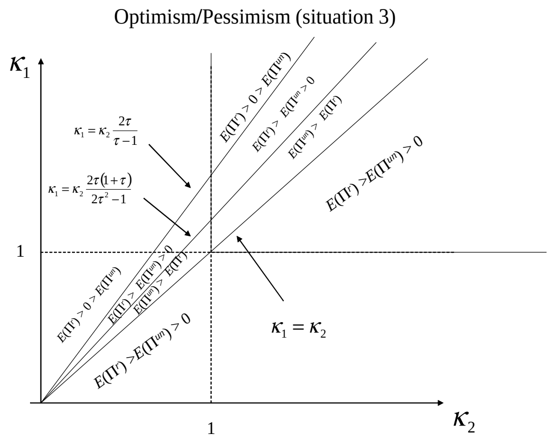

Figure 7: Expected Profit comparison for both a large τ $(1 < \tau)$ and a relatively low number of unrealistic traders in the market $(\frac{M}{N} \leq \frac{1+\tau}{(1-\tau)(1+2\tau)^2})$, with optimists/pessimists.