

Stochastic Portfolio Theory

Financial Models based on Information Theory

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Introduction

Stochastic Portfolio Theory is a descriptive theory of portfolio behavior, introduced by E.ROBERT FERNHOLZ,
Chief Investment Officer INTECH.

APPLICATIONS:

1. Comparison of Portfolio Strategies
2. Portfolio Optimization
3. Reaching Goals with given probability
4. ...

General Formulation

ASSETS: $X_i(t)$

PORTFOLIO: $\pi_i(t)$, $\sum_i \pi_i(t) = 1$

WEALTH = Portfolio Value = $Z_\pi(t)$

COMPARISON OF PORTFOLIOS

$$\log \frac{Z_\pi(t)}{Z_\eta(t)}$$

Relative Return of Portfolio π versus η .

Market Portfolio

PORTFOLIO WITH WEIGHTS

$$\mu_i(t) = \frac{X_i(t)}{X_1(t) + \cdots + X_n(t)}$$

The corresponding wealth is

$$Z_\mu(t) = X_1(t) + \cdots + X_n(t)$$

It represents a canonical benchmark.

DIVERSITY OF A MARKET

$$\mu_{\max}(t) \leq 1 - \delta, \quad \delta > 0.$$

At no time a single stock accounts for almost the entire market capitalization.

Entropy weighted portfolio

$$\pi_i(t) = -\frac{\mu_i(t) \log \mu_i(t)}{S(\mu(t))}$$

$$S(\mu(t)) = -\sum_i \mu_i(t) \log \mu_i(t)$$

S Entropy Function

$\mu(t)$ is the market portfolio.

The entropy weighted portfolio is less concentrated than the market portfolio in those stocks with the highest market weights.

Main Result

THEOREM 1. *For a non-degenerate diverse market*

$$\frac{Z_{\pi}(T)}{Z_{\mu}(T)} > \frac{Z_{\pi}(0)}{Z_{\mu}(0)}$$

for T sufficiently large.

The entropy-weighted portfolio has return above that of the market portfolio.

Dividend payments by the larger stocks can correct this property.

Comments about Information

1. Information is the driving force of decisions
2. Information Theory(Shannon) is more a theory of communication. One counts bits of information; contents are irrelevant.
3. External information is common to all players:interpretation of this information makes the difference.
4. Internal information is not common. Portfolios concentrated on a limited number of stocks need internal information to perform well.

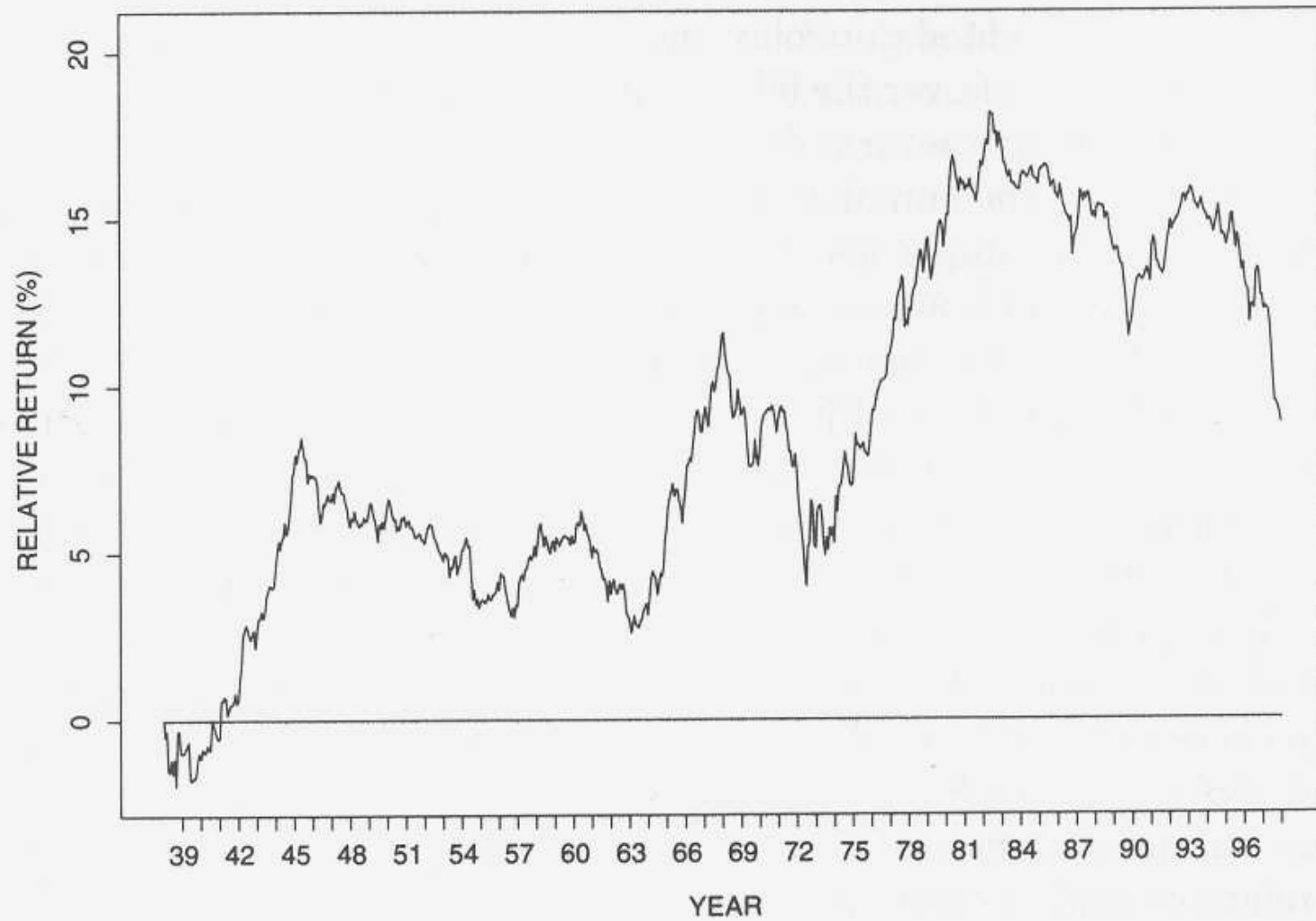


FIGURE 6.1. $\log(\hat{Z}_\pi(t)/\hat{Z}_\mu(t))$ for the entropy-weighted portfolio.